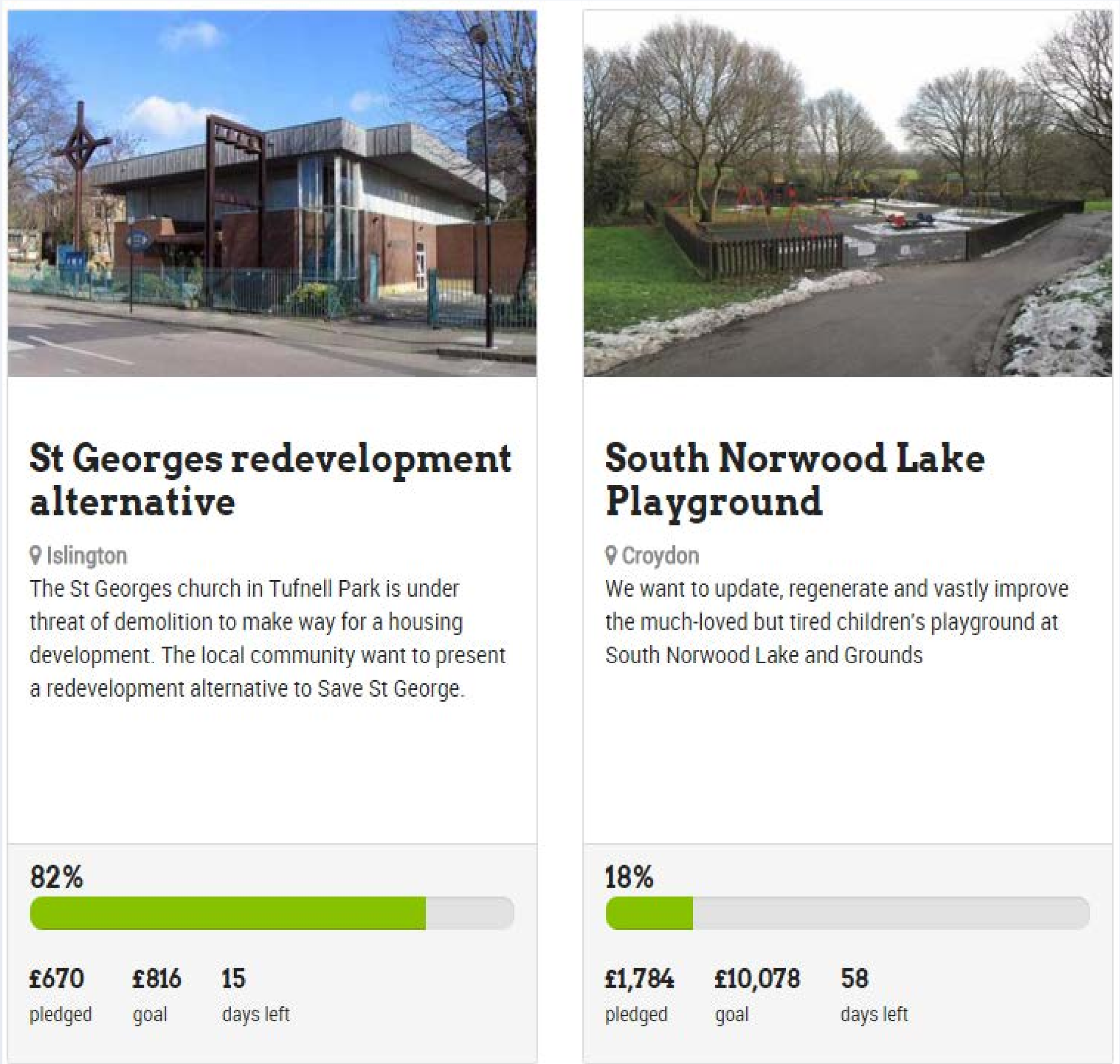
Crowdfunding Public Projects with Provision Point: A Prediction Market Approach

Praphul Chandra, Sujit Gujar, Y. Narahari
Indian Institute of Science, Bangalore, India.

Motivation

Crowdfunding | Private Provisioning of Public Goods



Crowdfunding Process

1. Requester posts public project (non-excludable)
2. Agents arrive & observe :
   a) target amount (provision point),
   b) deadline
   c) pending amount.
3. Agents contribute (or not)
4. Requester executes project or refunds.

Mechanism Design Motivation

Agent's true value for the project is private info.
Strategic agents can freeride (No/Low contribution).
Strategic agents can delay contribution.
Project may not be funded even if everyone values it!

Mechanism Design: Induce a game s.t. agents contribute

Related Work

1. [Bagnoli & Lipman '89] : Provision Point Mechanism
   a) Simultaneous move game
   b) Project not funded at multiple equilibria.
2. [Zubrickas '14] : PPM with Refund bonus
   a) Simultaneous move game
   b) Project funded at equilibria.
3. [Our work] : PPM with Securities
   a) Sequential game
   b) Subgame perfect equilibria: project funded.
   c) Agents contribute in proportion to value
   d) Agents contribute as soon as they arrive
4. [Hanson'03], [Chen & Pennock '10]: Prediction Mkt
   a) Software agents: securities for prediction.
   b) Scoring Rule ← →Cost Function.
   c) Specially suited for thin markets.

Mechanism Design

How to incentivize private citizens to contribute to public projects? The Freeriding problem.

Table 1: Key Notation

Symbol	Definition
T	Time at which fund collection ends
h^t	Amount that remains to be funded at t; h^0 is the target amount
$i \in \{0, 1, \ldots, n\}$	Agent id; $i = 0$ refers to the requester
$\theta_i \in \mathbb{R}_+$	Agent i's value for the project
$x_i \in \mathbb{R}_+$	Agent i's contribution to the project
$a_i \in [0, T]$	Time at which agent i arrives at the platform
$t_i \in [a_i, T]$	Time at which agent i makes a contribution towards the project
$\psi_i = (x_i, t_i)$	Strategy of agent i
$\vartheta \in \mathbb{R}_+$	Net value for the project
$\chi \in \mathbb{R}_+$	Net contribution for the project
$k \in \{0, 1\}$	Project provisioning decision

$$u_i(\psi;\theta_i) = \mathcal{I}_{\chi \ge h^0} \times (\theta_i - x_i) + \mathcal{I}_{\chi < h^0} \times (r_i^{t_i} - x_i)$$

Intuitive Explanation

1. Incentivizes agents to contribute by offering them a bonus greater than their contribution.
2. Bonus paid out ***iff*** the project is not funded.
3. Ensures that project is funded at equilibrium.

Novel Idea: Use prediction markets for bonus!

Provision Point with Securities

Binary Event: At deadline, project funded or not ?
Positive securities pay $1 if project funded.
Negative securities pay $1 if project is not funded.
Software agent always accepts trades.
Price as first order derivative of cost function.

$$C_{LMSR}(\mathbf{q}) = b\ln(\exp(q_{\omega_0}/b) + \exp(q_{\omega_1}/b))$$

Prediction Market issues only Negative securities

$$C_0(q^t) = b\ln(1 + \exp(q^t/b))$$

Number of securities issued to an agent depend on
a) Quantum of his contribution
b) Timing of his contribution

$$r_i^{t_i} = b\ln\left(\exp\left(\frac{x_i}{b} + \ln(1 + \exp(\frac{q^{t_i}}{b}))\right) - 1\right) - q^{t_i}$$

Software agent (sponsor) pays out only if project is not funded.

PPS Equilibrium

If
- Net value of the project > Cost of the project
- b є (0, (ϑ – h⁰)/ ln 2)

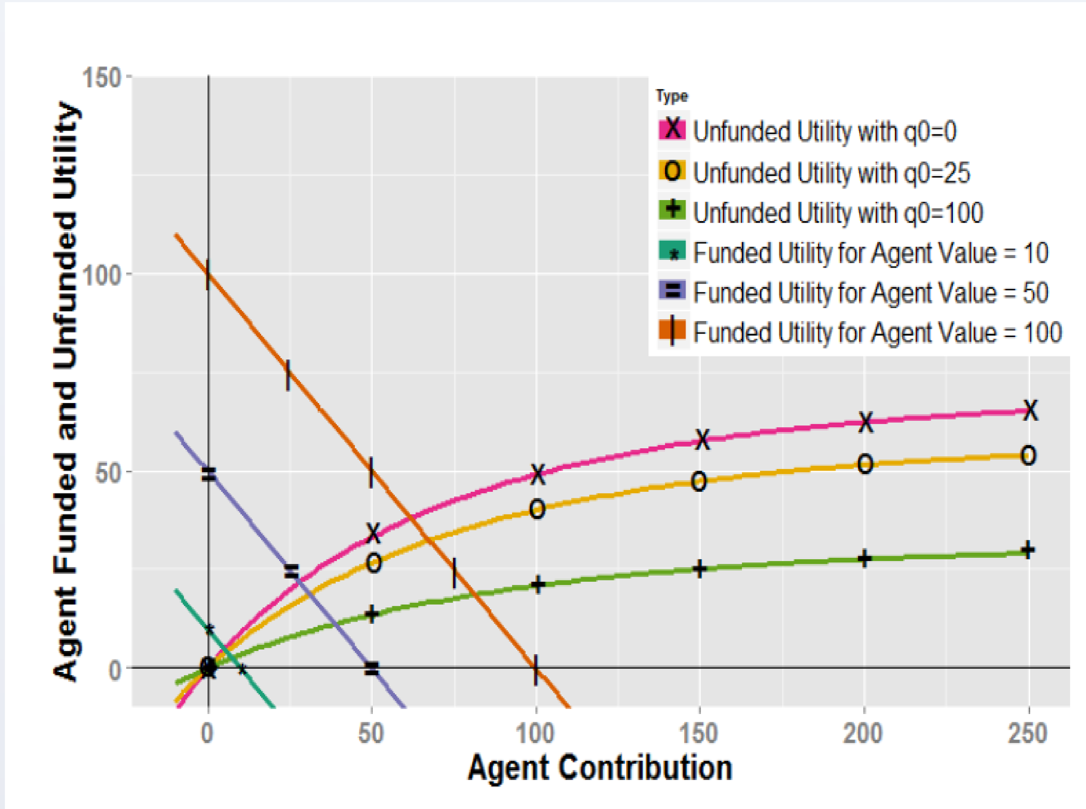
Then
- Project is funded at Equilibrium
- Equilibrium is subgame perfect (sequential game)
- Each agent contributes in proportion to his value
- Each agent contributes as soon as he arrives
- Agents have an incentive to contribute early.

$$x_i^* \le C_0(\theta_i + q^{a_i}) - C_0(q^{a_i}) = b\ln\left(\frac{1 + \exp\left(\frac{\theta_i + q^{a_i}}{b}\right)}{1 + \exp\left(\frac{q^{a_i}}{b}\right)}\right)$$

Equilibria are subgame perfect (Sequential Game)

LMSR-PPS

Leverage infinite liquidity of LMSR to create a prediction market where each agent has an incentive to contribute.



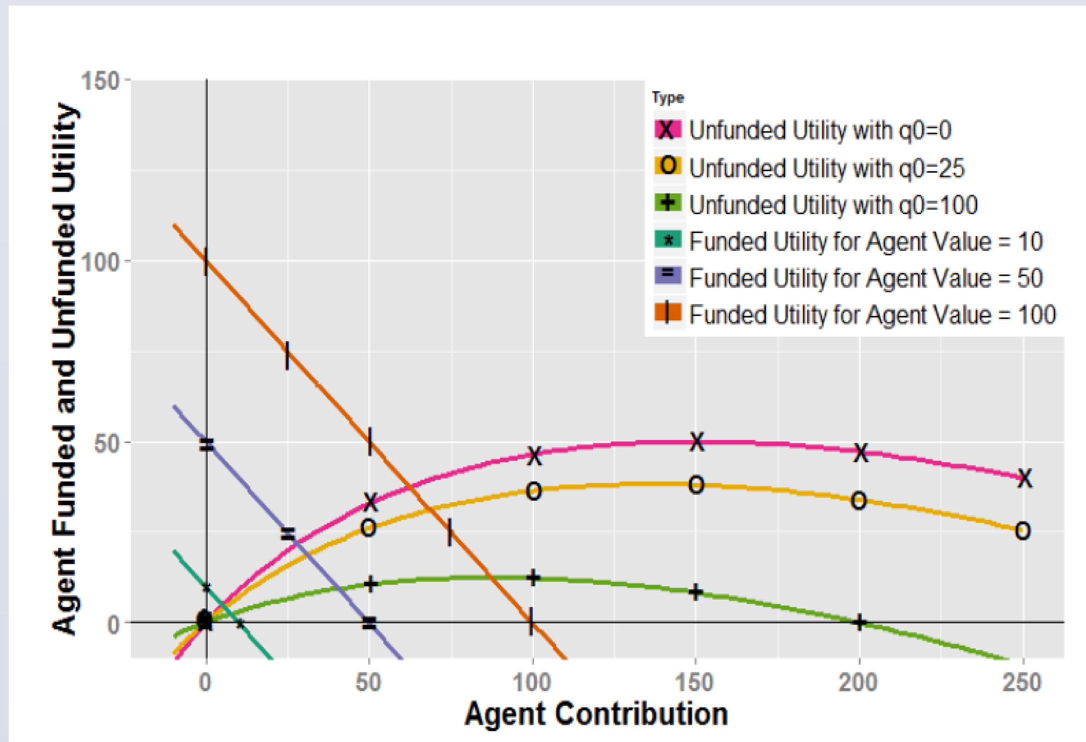
$$u_i(\psi;\theta_i) = \mathcal{I}_{\chi \ge h^0} \times (\theta_i - x_i) + \mathcal{I}_{\chi < h^0} \times (r_i^{t_i} - x_i)$$

Funded Utility
a) Monotonically decreases with contribution

Unfunded Utility
a) Monotonically increases with contribution
b) Monotonically decreases with outstanding securities (time)

QSR-PPS

Other cost functions can be used if parameterized correctly.



Necessary conditions on Cost Function

1. Path Independence $\texttt{Cost}(\mathbf{r}|\mathbf{q}) = C(\mathbf{q}+\mathbf{r}) - C(\mathbf{q})$
2. Continuous & Differentiable $p_{\omega_j} = \partial C(\mathbf{q})/\partial(q_{\omega_j}) \ge 0 \quad \forall \omega_j \in \Omega$
3. Information Incorporation $C(\mathbf{q}+2\mathbf{r}) - C(\mathbf{q}+\mathbf{r}) \ge C(\mathbf{q}+\mathbf{r}) - C(\mathbf{q})$
4. No Arbitrage $\exists \omega_j \in \Omega$ such that $C(\mathbf{q}+\mathbf{r}) - C(\mathbf{q}) > \mathbf{r} \cdot \pi_{\omega_j}$
5. ~~Expressiveness~~ $\forall \mathbf{p} \in \Delta_{|\Omega|}, \exists \mathbf{q} \in \mathbb{R}^{|\Omega|}$ s.t. $\nabla C(\mathbf{q}) = \mathbb{E}_{\omega\sim\mathbf{p}}[\pi(\omega)]$
6. Bounded Loss $\sup_{\mathbf{q}}[\max_{\omega_j}(q_{\omega_j}) - C(\mathbf{q})] < \infty$.
7. Sufficient Liquidity $\forall q^{t_i}, \forall x_i < h^0, \quad \frac{\partial}{\partial x_i}(r_i^{t_i} - x_i) > 0 \Rightarrow \frac{\partial r_i^{t_i}}{\partial x_i} > 1$.

References

[Bagnoli and Lipman, 1989] Mark Bagnoli and Barton Lipman. Provision of public goods: Fully implementing the core through private contributions. The Review of Eco-nomic Studies, 56(4):583-601, 1989.
[Hanson, 2003] Robin Hanson. Combinatorial informationmarket design. Information Systems Frontiers, 5(1):107-119, 2003.
[Chen and Vaughan, 2010] Yiling Chen and Jennifer Wortman Vaughan. A new understanding of prediction mar-kets via no-regret learning. In Proceedings of the 11th ACM conference on Electronic commerce, pages 189-198. ACM, 2010.
[Othman et al., 2013] Abraham Othman, David M Pennock, Daniel M Reeves, and Tuomas Sandholm. A practical liquidity-sensitive automated market maker. ACM Trans-actions on Economics and Computation, 1(3):14, 2013.
[Zubrickas, 2014] Robertas Zubrickas. The provision point mechanism with refund bonuses. Journal of Public Eco-nomics, 120:231-234, 2014.