Modeling and Analysis of Networks with High-speed TCP Connections

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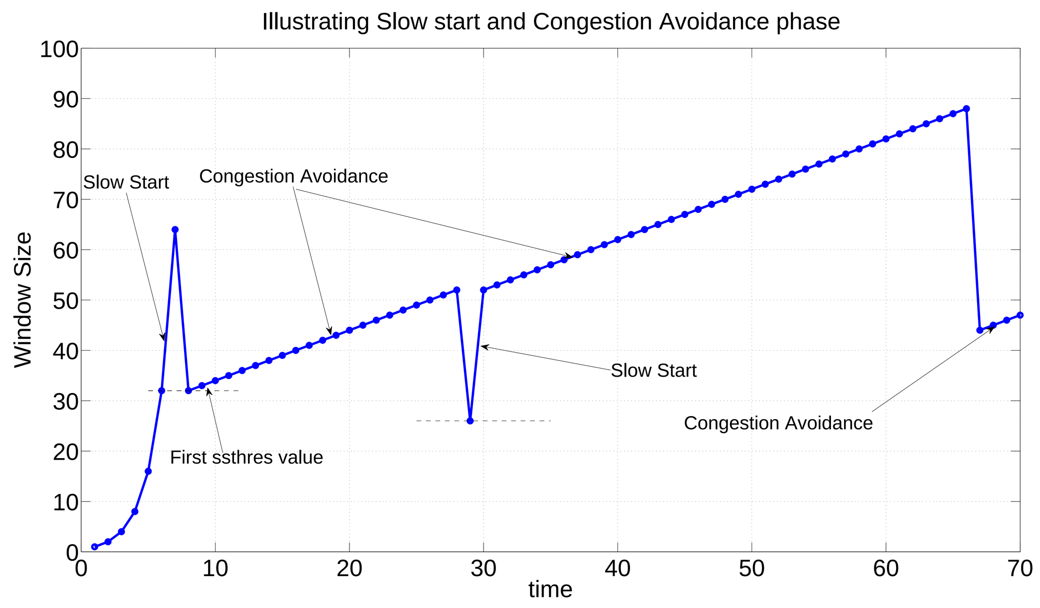
Traditional TCP

Transmission Control Protocol (TCP) is a dominant transport layer protocol used over the Internet, which provides reliable, in-order end-to-end data transfer.
It also does congestion control to prevent network congestion and flow control to avoid overwhelming the receiver.
We focus on congestion control. TCP congestion control has two phases:

- **Slow start**, $W_n < ssthres$
- **Congestion avoidance**, $W_n \geq ssthres$



AIMD TCP congestion control prevents congestion and is 'fair'. However, $\cdots$

- TCP does not distinguish between non-congestion losses and congestion losses.
  - **poor performance in wireless environment**.
- For high speed networks, AIMD TCP is too slow.
  - **inefficient link usage in large BDP networks**.

High Speed TCP

High speed TCP algorithms use **aggressive window updates**. We study **TCP CUBIC** and **TCP Compound** as they are **widely used**.

TCP CUBIC

Default Linux TCP algorithm since 2006.
TCP CUBIC Window Evolution:

$$W_{cubic}(W_0, t) = \mathcal{C}(t - \sqrt[3]{(W_0\beta/\mathcal{C})})^3 + W_0. \quad (1)$$

- W_0: window size at the last loss epoch
- t: time since last loss; β: the multiplicative drop factor
- If loss, the window size is reduced by a factor of $(1-\beta)$.

Also uses

$$W_{reno}(W_0, t) = W_0(1-\beta) + 3\frac{\beta}{2-\beta}\frac{t}{R}. \quad (2)$$

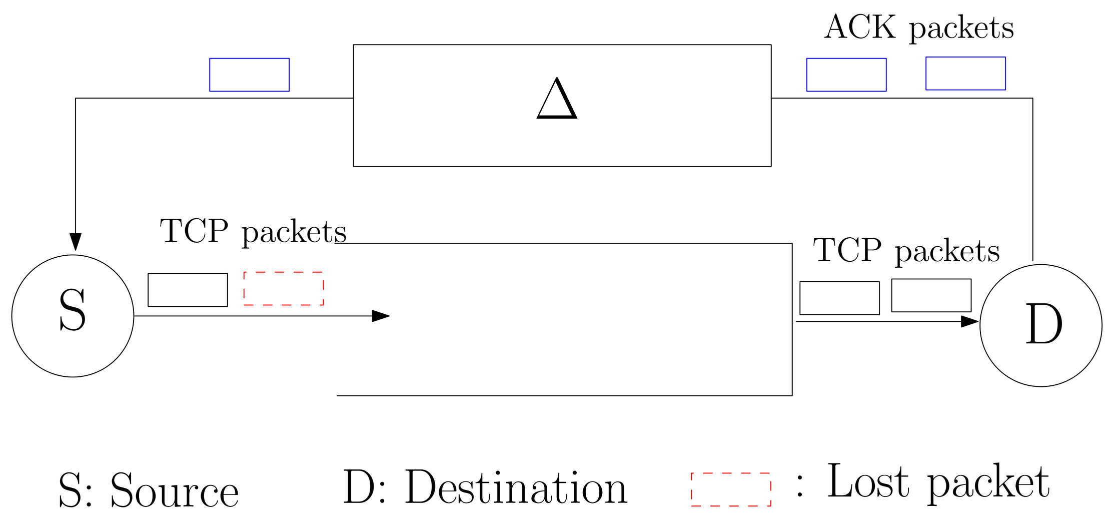
TCP Compound

- It is used by Windows servers.
- W_n : Window size at end of n^{th} RTT.
- The TCP Compound window size is given by

$$W_{n+1} = \begin{cases} W_n + \alpha W_n^k, & \text{if no loss} \\ \frac{W_n}{2}, & \text{if loss is detected;} \end{cases} \quad (3)$$

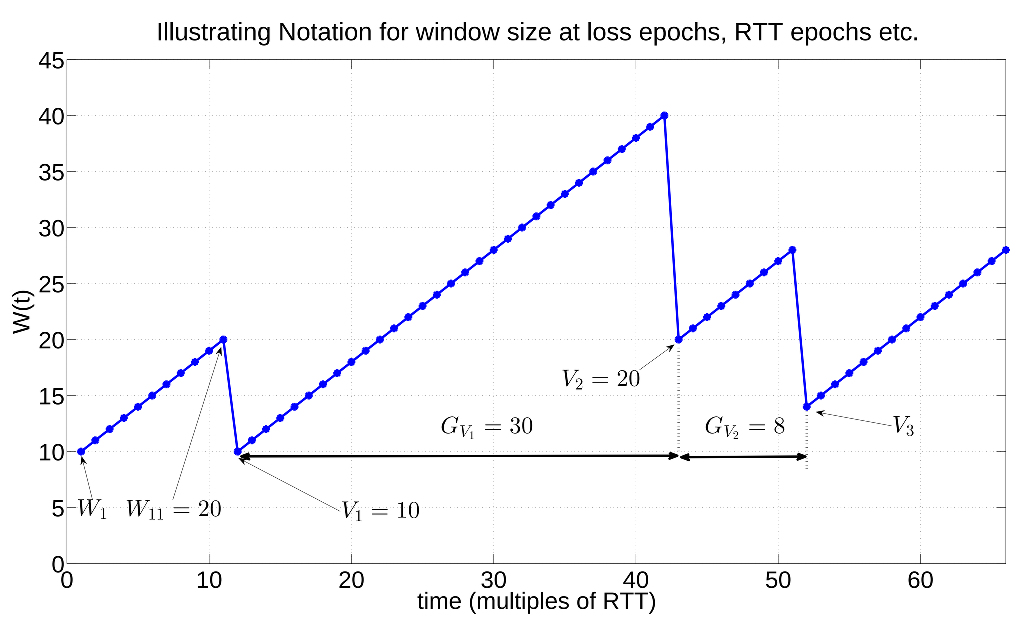
System Model

We have a single long-lived TCP flow with constant RTT.
Each packet of the flow is dropped w.p. p independently of the other packets.



Our Contribution

- The assumption of random losses is reasonable in wireless networks.
- The Markov models are more exact than fluid models but do not provide closed form solutions.
- We derive a closed-form approximation for TCP throughput under random losses for TCP CUBIC and TCP Compound.



- $W_n(p)$: Window size at the end of n^{th} RTT.
- $V_k(p)$: Window size at the end of k^{th} loss epoch.
- $G^p_{V_k(p)}$: Time between the k^{th} and the $(k+1)^{st}$ loss epochs.

Stationarity of $\{W_n(p)\}$ process

We show that the window size process, $\{W_n(p)\}$ has a unique stationary distribution and has finite mean under stationarity for TCP CUBIC and TCP Compound.

Asymptotic Approximations

As $p \to 0$, $W_n(p) \to \infty$. However, if we consider an appropriately scaled version of $\{W_n(p)\}$, we can derive some useful results.
For the time between losses, we have

- TCP Comp.: For $x \geq 1$, $p^{\frac{1-k}{2-k}} G^p_{\lfloor \frac{x}{p^{\frac{1}{2-k}}} \rfloor} \xrightarrow{w} \overline{G}_x$, as $p \to 0$, with $\mathbb{P}(\overline{G}_x \geq y) = f_{ctcp}(\alpha, k, x, y)$.
- TCP CUBIC: For $x \geq 1$, $p^{\frac{1}{4}} G^p_{\lfloor \frac{x}{p^{\frac{3}{4}}} \rfloor} \xrightarrow{w} \overline{G}_x$, as $p \to 0$, where $\mathbb{P}(\overline{G}_x \geq y) = f_{cubic}(C, R, x, y)$.

For the $\{V_k(p)\}$ process, (with $\{\overline{V}_k\}$, a Markov process with transitions dependent on $\overline{G}_{\overline{V}_{k-1}}$), as $p \to 0$,

- TCP Comp.: If $\lim_{p\to 0} p^{\frac{1}{2-k}} V_0(p) \xrightarrow{w} \overline{V}_0$, $\{p^{\frac{1}{2-k}} V_n(p)\} \xrightarrow{w} \{\overline{V}_n\}$.
- TCP CUBIC: If $\lim_{p\to 0} p^{\frac{3}{4}} V_0(p) \xrightarrow{w} \overline{V}_0$, $\{p^{\frac{3}{4}} V_n(p)\} \xrightarrow{w} \{\overline{V}_n\}$.

Throughput Approximation

Now, by Little's law, $\mathbb{E}[W(p)] = \frac{\frac{1}{p}}{\mathbb{E}[G^p_{V(p)}]}$

- TCP Compound: By simulations we get, $\frac{1}{\mathbb{E}[\overline{G}_{\overline{V}_\infty}]} = 0.257$. Therefore for small p,

$$\mathbb{E}[W(p)] \approx 0.257 p^{-\frac{1}{2-k}}, \quad (4)$$

- TCP CUBIC: By simulations, we get $\frac{1}{\mathbb{E}[\overline{G}_{\overline{V}_\infty}]} = 1.3004$, for $R = 1$, Hence,

$$\mathbb{E}[W(p)] \approx \max\Big\{1.3004\Big(\frac{R}{p}\Big)^{\frac{3}{4}}, \frac{1.31}{\sqrt{p}}\Big\}. \quad (5)$$

Simulation Results

Table: TCP Compound: Mean window size

per (p)	$\mathbb{E}[W]$ Simulations (ns2)	$\mathbb{E}[W]$ Det. Fluid [2]	$\mathbb{E}[W]$ Markov chain [6]	$\mathbb{E}[W]$ Approx. Markov $0.2570p^{-\frac{1}{2-k}}$
1×10^{-2}	13.06	10.16	12.99	10.23
5×10^{-3}	19.16	17.69	19.06	17.81
3×10^{-3}	25.83	26.63	26.67	26.80
1×10^{-3}	58.96	64.12	63.46	64.54
8×10^{-4}	71.67	76.66	76.16	77.16
5×10^{-4}	108.30	111.65	111.68	112.38
3×10^{-4}	166.62	168.01	168.90	169.11
1×10^{-4}	414.97	404.60	409.26	407.25
8×10^{-5}	499.14	483.67	489.64	486.85
5×10^{-5}	727.09	704.45	714.25	709.07
3×10^{-5}	1115.37	1060.05	1076.51	1067.01

Table: TCP CUBIC: Mean window size

per p	RTT R (sec)	$\mathbb{E}[W]$ Simulations (ns2)	$\mathbb{E}[W]$ Det. Fluid [1]	$\mathbb{E}[W]$ Markov chain [4]	$\mathbb{E}[W]$ Approx. Markov Eqn. (5)
1×10^{-2}	1	39.97	**33.33**	37.44	**41.19**
1×10^{-2}	0.2	14.3	13.10	13.53	13.10
1×10^{-2}	0.1	12.62	13.10	12.50	13.10
1×10^{-2}	0.02	12.08	13.10	12.41	13.10
5×10^{-3}	1	69.46	**56.05**	63.78	**69.27**
5×10^{-3}	0.2	21.82	18.53	21.02	**20.81**
5×10^{-3}	0.1	18.29	18.53	18.09	18.53
5×10^{-3}	0.02	17.21	18.53	17.73	18.53
1×10^{-3}	1	229.96	**187.40**	218.32	**231.63**
1×10^{-3}	0.2	67.83	**56.05**	67.92	**69.58**
1×10^{-3}	0.1	44.68	41.43	44.55	41.43
1×10^{-3}	0.02	39.40	41.43	39.94	41.43
5×10^{-4}	1	384.43	**315.17**	370.12	**388.56**
5×10^{-4}	0.2	113.05	**94.26**	114.52	**117.02**
5×10^{-4}	0.1	69.12	58.58	70.05	**69.24**
5×10^{-4}	0.02	55.89	58.58	56.66	58.58
8×10^{-5}	1	1507.19	**1245.81**	1487.19	**1539.87**
8×10^{-5}	0.2	430.49	**372.58**	454.41	**462.57**
8×10^{-5}	0.1	260.91	**221.54**	271.15	**273.69**
8×10^{-5}	0.02	143.99	146.46	143.42	146.46

Summary

- We derive throughput approximations for TCP CUBIC and TCP Compound under random losses via analytical models.
- Our model approximations have been validated by ns2 simulations.

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