An Alternating Projections Algorithm for Sparse Blind Deconvolution: Application to Speech Signal Deconvolution

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1. Problem Formulation

Consider the linear measurement model

$$\mathbf{y} = \mathbf{h} * \mathbf{e} + \mathbf{w} = \mathbf{H}\mathbf{e} + \mathbf{w} = \mathbf{E}\mathbf{h} + \mathbf{w},$$

$\mathbf{h} \in \mathbb{R}^L$: point spread function (PSF), $\mathbf{e} \in \mathbb{R}^M$: Excitation, $\mathbf{y} \in \mathbb{R}^N$: observation, $\mathbf{w} \in \mathbb{R}^N$: Noise $(N = L + M - 1)$. $\mathbf{E} \in \mathbb{R}^{N\times L}$, $\mathbf{H} \in \mathbb{R}^{N\times M}$ are linear convolution matrices (Conv. mtx.) corresponding to $\mathbf{e}$ and $\mathbf{h}$, respectively.

- **Objective:** Estimate $\mathbf{h}$ and $\mathbf{e}$ from $\mathbf{y}$.
  - $\mathbf{e}$ is assumed to be a sparse vector and $\mathbf{h}$ is a smooth and stable operator.
  - The focus is on developing a BD algorithm for such cases.

2. Issues

- Infinitely many solutions to (1). Hence, we need priors on $\mathbf{e}$ and $\mathbf{h}$ to reduce search space.
- The formulated cost function needs to be optimized over both $\mathbf{e}$ and $\mathbf{h}$ and is hence non-convex, thus leading to local minima issues.
- Need a good initialization to avoid sub-optimal local minima.

3. Our Contribution

- We formulate the SBD problem using the MAP formulation and propose an alternating minimization algorithm to optimize the resulting cost function.
- We show that for well-conditioned systems the pseudo-inverse solution is a good initialization.
- We analyze the convergence property of the algorithm.
- We show application to epoch estimation in natural speech signals.

4. MAP Formulation for BD problem

- $\mathbf{h}$: deterministic but unknown.
- $\mathbf{e}$: Entries e_i are assumed to be *i.i.d. generalized p-Gaussian distribution* (gpG).

$$f(\mathbf{e}) = \left(\frac{p}{2\Gamma(1/p)\gamma\sigma_e}\right)^M \exp\left(-\sum_i \left(\frac{|e_i|}{\gamma\sigma_e}\right)^p\right),$$

where $\gamma = \left(\frac{\Gamma(1/p)}{\Gamma(3/p)}\right)^{1/2}$ and $0 \leqslant p \leqslant 1$.

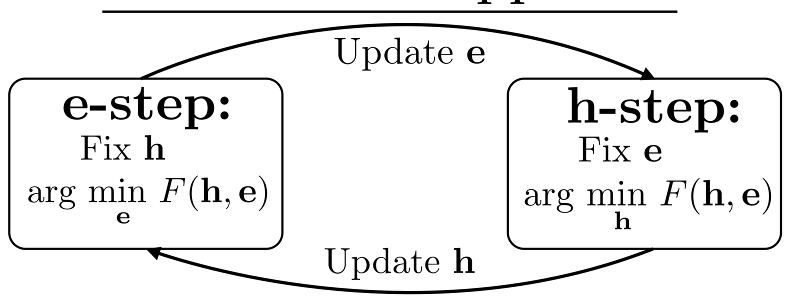
- The MAP estimates of the vectors $\mathbf{h}$ and $\mathbf{e}$ are

$$(\mathbf{h}_{\text{MAP}}, \mathbf{e}_{\text{MAP}}) = \arg\max_{\mathbf{h},\mathbf{e}} f(\mathbf{y}; \mathbf{e}; \mathbf{h}) f(\mathbf{e}),$$
$$= \arg\min_{\mathbf{h},\mathbf{e}} \underbrace{\|\mathbf{y} - \mathbf{H}\mathbf{e}\|_2^2 + \lambda\|\mathbf{e}\|_p^p}_{F(\mathbf{h},\mathbf{e})}.$$

The joint cost $F(\mathbf{h}, \mathbf{e})$ is non-convex and not straightforward to optimize.

5. An Alternating $\ell_p - \ell_2$ Projections Algorithm (ALPA)

The Alt. Min. approach:



e-step: $\mathbf{e}^{(k+1)} = \arg\min_{\mathbf{e}} F\left(\mathbf{h}^{(k)}, \mathbf{e}\right) = \arg\min_{\mathbf{e}} \|\mathbf{y} - \mathbf{H}^{(k)}\mathbf{e}\|_2^2 + \lambda\|\mathbf{e}\|_p^p,$

where $\mathbf{H}^{(k)} \in \mathbb{R}^{N\times M} = \text{Conv. mtx.}(\mathbf{h}^{(k)})$.

- For $0 \leqslant p < 1$, $F(\mathbf{h}^{(k)}, \mathbf{e})$ is non-convex.
- $\nabla_{\mathbf{e}} F\left(\mathbf{h}^{(k)}, \mathbf{e}\right)$ has a singularity at $\mathbf{e} = \mathbf{0}$.

5. An Alternating $\ell_p - \ell_2$ Projections Algorithm (ALPA) (contd.)

Iteratively reweighted least-squares (IRLS)

Majorize $\|\mathbf{e}\|_p^p$ with weighted ℓ_2-norm function and minimize the cost iteratively.

$$\tilde{\mathbf{e}}^{(j+1,k)} = \arg\min_{\mathbf{e}} \underbrace{\|\mathbf{y} - \mathbf{H}^{(k)}\mathbf{e}\|_2^2 + \lambda\mathbf{e}^T\mathbf{W}^{j,k}\mathbf{e}}_{F_\epsilon(\mathbf{h}^{(k)},\mathbf{e})},$$

$$\text{where } \mathbf{W}^{(j,k)} = \text{diag}\left(p\left((\tilde{e}_i^{(j,k)})^2 + \epsilon\right)^{p/2-1}\right).$$

Final solution of e-step (after J iterations of IRLS): $\mathbf{e}^{(k+1)} = \tilde{\mathbf{e}}^{(J,k)}$,

$$\mathbf{e}^{(k+1)} = \left((\mathbf{H}^{(k)})^T(\mathbf{H}^{(k)}) + \lambda\mathbf{W}^{(J,k)}\right)^{-1}(\mathbf{H}^{(k)})^T\mathbf{y},$$

Use $\mathbf{e}^{(k+1)}$ to update $\mathbf{h}^{(k+1)} = \arg\min_{\mathbf{h}} F_\epsilon\left(\mathbf{h}, \mathbf{e}^{(k+1)}\right)$.

h-step: $\tilde{\mathbf{h}}^{(k+1)} = \arg\min_{\mathbf{h}} \|\mathbf{y} - \mathbf{E}^{(k+1)}\mathbf{h}\|_2^2 = \mathbf{E}^{(k+1)\dagger}\mathbf{y},$

$$\mathbf{h}^{(k+1)} = \tilde{\mathbf{h}}^{(k+1)}/\|\tilde{\mathbf{h}}^{(k+1)}\|_2^2.$$

6. Initialization: Pseudo-inverse Solution

- Given some initial filter estimate $\tilde{\mathbf{h}}$, how good is the least-squares solution (obtained by setting $\mathbf{W}^{(0,0)} = \mathbf{0}$) as initialization?
- If true excitation and filter are $\mathbf{e}^*$ and $\mathbf{h}^*$ respectively, $\delta\mathbf{h} = \mathbf{h}^* - \tilde{\mathbf{h}}$ and pseudo-inverse solution is denoted $\hat{\mathbf{e}}_{\text{BLS}}$. The difference

$$\delta\mathbf{e}_{\text{BLS}} = \mathbf{e}^* - \hat{\mathbf{e}}_{\text{BLS}} = \mathbf{H}^{*\dagger}(\mathbf{w} - \delta\mathbf{H}\mathbf{e}^*),$$

where $\delta\mathbf{H} = \text{Conv. mtx}(\delta\mathbf{h})$.

- What is the probability of the average absolute error $\frac{1}{M}\|\delta\mathbf{e}_{\text{BLS}}\|_1$ becoming too large a value ξ?

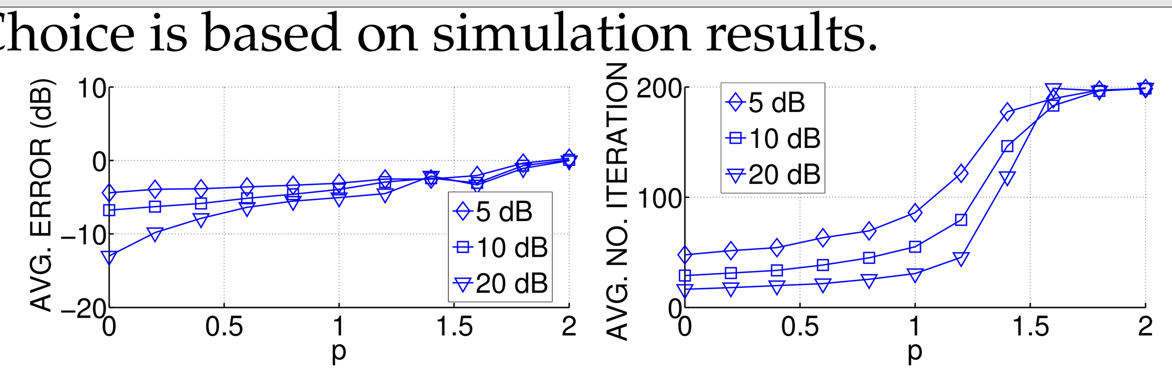
$$P\left(\frac{1}{M}\|\delta\mathbf{e}_{\text{BLS}}\|_1 > \xi\right) \leqslant \frac{\kappa^2}{(\xi - \kappa\|\delta\mathbf{H}\|_2\|\mathbf{e}^*\|_2/\sqrt{M})^2}.$$

κ is the condition number of the linear system.

- Bound applicable only when $\xi > \kappa\|\delta\mathbf{H}\|_2\|\mathbf{e}^*\|_2/\sqrt{M}$ (comes for Markov inequality).

7. Choice of 'p'

- Choice is based on simulation results.



- Roll-off in the excitation estimation error or number of iteration was not significant in the range $0 \leqslant p \leqslant 0.5$, we chose $p = 0.1$.

7. Convergence Guarantees

- After **e-step**,

$$F_\epsilon(\mathbf{h}^{(k)}, \mathbf{e}^{(k+1)}) \leqslant F_\epsilon\left(\mathbf{h}^{(k)}, \mathbf{e}^{(k)}\right).$$

- Similarly, after **h-step**,

$$F_\epsilon(\mathbf{h}^{(k+1)}, \mathbf{e}^{(k)}) \leqslant F_\epsilon\left(\mathbf{h}^{(k)}, \mathbf{e}^{(k)}\right).$$

- After one cycle of ALPA, the majorized cost function $F_\epsilon\left(\mathbf{h}^{(k)}, \mathbf{e}^{(k)}\right)$

$$F_\epsilon(\mathbf{h}^{(k+1)}, \mathbf{e}^{(k+1)}) \leqslant F_\epsilon\left(\mathbf{h}^{(k)}, \mathbf{e}^{(k)}\right).$$

- Further, with ϵ updated as

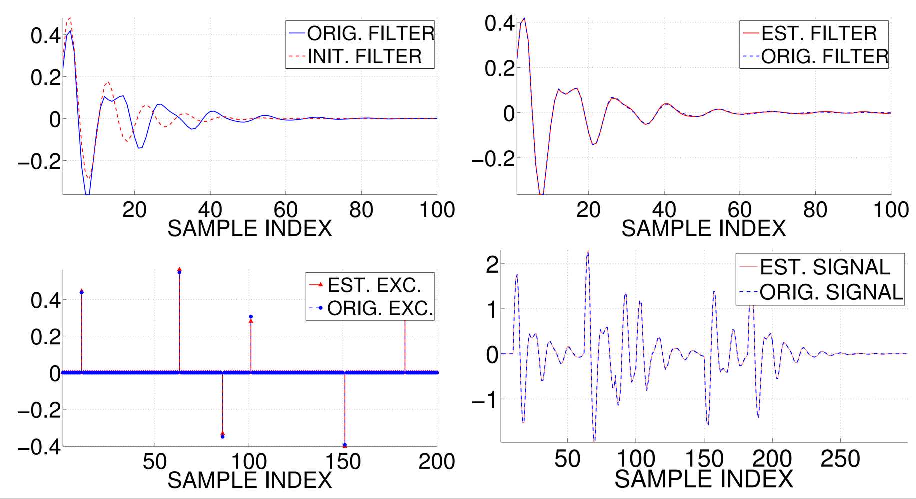
$$\epsilon^{(k)} = c\left(\max\left|e_j^{(k)}\right|\right)^{2-p}, \quad \text{with } 0 < c \ll 1, \text{ actual}$$

cost function $F\left(\mathbf{h}^{(k)}, \mathbf{e}^{(k)}\right)$ is also non-increasing,

$$F\left(\mathbf{h}^{(k+1)}, \mathbf{e}^{(k+1)}\right) \leqslant F\left(\mathbf{h}^{(k)}, \mathbf{e}^{(k)}\right).$$

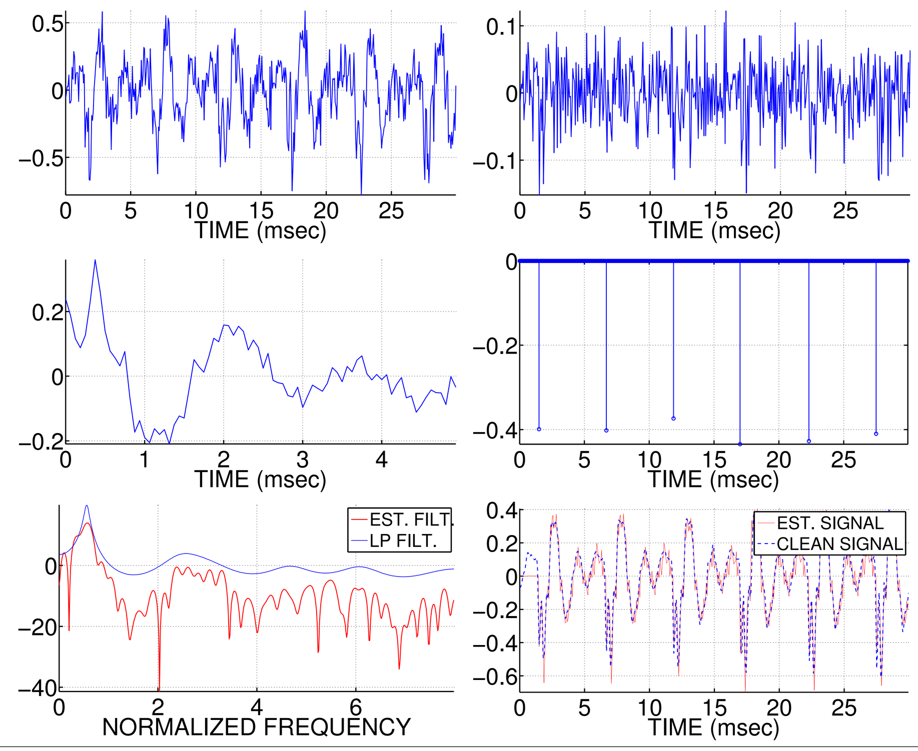
8. Simulation Results

- Filter $h(n) = \sum_{k=1}^{3} e^{-\alpha_k n}\cos(\omega_k n)\, u(n),\ n = 1, 2, 3, \cdots, L,$



9. Voiced speech signal

- 30 ms vowel segment /æ/ (female speaker)



10. Comparisons

- Comparions with Smooth ℓ_1/ℓ_2 blind deconvolution (SOOT) [1], sparse linear prediction (SLP) [2] and MM-based Sparse Deconvolution (SDMM) [3].

Noise standard deviation →		0.01	0.02	0.03
MSE in excitation (dB)	ALPA	−17.4	−10.7	−8.3
	SOOT	−2.9	−2.8	−2.8
	SLP	−0.04	0.46	0.74
	SDMM	−22.6	−11.0	−3.7
MSE in filter (dB)	ALPA	−15.0	−14.3	−10.0
	SOOT	−10.5	−10.1	−9.3
	SLP	−4.3	−3.1	−2.7
	SDMM	−10.9	−6.0	−4.1
MSE in reconstruction (dB)	ALPA	−21.6	−17.5	−14.1
	SOOT	−25.24	−20.1	−16.8
	SLP	−15.6	−12.0	−9.2
	SDMM	−12.5	−6.8	−4.6
Average time (sec.)	ALPA	0.1	0.1	0.2
	SOOT	1.3	1.3	1.3
	SLP	2.7	2.8	2.8
	SDMM	0.16	0.17	0.17

11. References

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