Incentivized Campaigning in Social Networks

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Motivation

- Social recommendation of products and services is very useful for enlarging the market share.
- Individuals are incentivized to recommend services to their friends and acquaintances.
- Uber, Lyft give free rides for recommending the app, Living Social gives discounts if recommendations lead to purchase.
- ✗ Referrals come at a big price. Giving referral incentives to everyone may not be a good idea.
- Provide referral incentives to only a few individuals that are capable of spreading the campaign.

The Problem and Proposed Solution

- **Marketing campaign for a freemium service.**
- **Goal : Maximize registrations.**
  - Maximize registrations (campaign size) for a given cost budget.
  - Achieve a given marketing target with minimum cost.
- **Solution : Probabilistic Incentivization**
  - Incentivize individuals with degree k with probability $\phi(k)$
  - Incentives provided to individuals for recommending service to friends.
  - Provide incentives only if they register.

Campaign Spreading Model

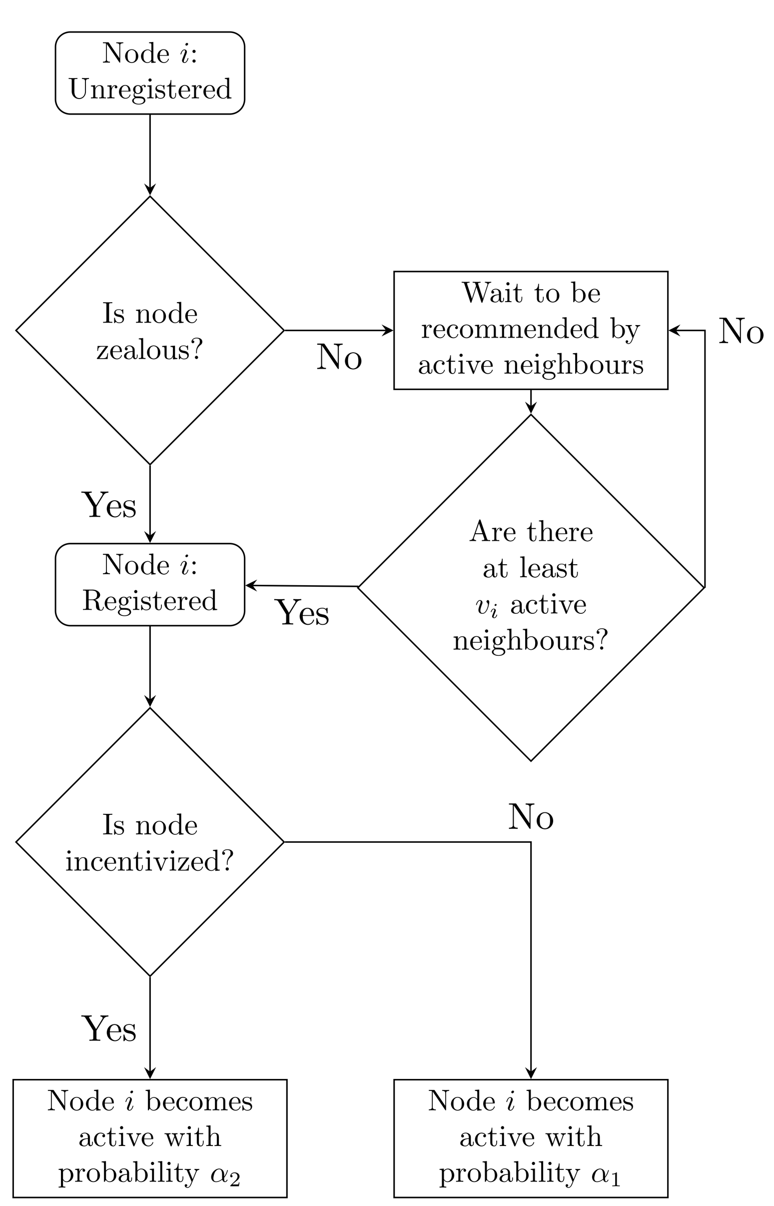

Figure 1: Flow chart denoting the various stages of a node; v_i denotes a deterministic *threshold* of node i, and α_1 and α_2 are the probability of activation of the *incentivized* and *non-incentivized* nodes, respectively.

Network Assumptions

- **Uncorrelated network**
  - Probability that an edge exists between two randomly chosen nodes is independent of the properties (such as degree) of the nodes.
- **Locally tree like network**

Optimization Problem Formulation and Main Results

Problem Formulation

$$\min_{0\le\phi\le1} \sum_{k\ge1} c_k\cdot p(k)\cdot\phi(k)\cdot s_k(q)$$

Subject to:

$$s(q)\ge\gamma$$

$$q=\frac{1}{\bar{d}}\sum_{k\ge1} k\cdot p(k)\cdot\phi(k)$$

$$\max_{0\le\phi\le1} s(q)$$

Subject to:

$$\sum_{k\ge1} c_k\cdot p(k)\cdot\phi(k)\cdot s_k(q)\le\bar{c}$$

$$q=\frac{1}{\bar{d}}\sum_{k\ge1} k\cdot p(k)\cdot\phi(k)$$

Size of the campaign

- $s(q)$ is the probability that a randomly chosen node registers for a campaign.
- $s_k(q)$ is the probability that a node with k neighbors registers for the campaign
- $p(k)$ is the degree distribution.
- $\bar{d}$ is the mean degree.
- $s(q)=g(q,u)|_{u=u_q}$ and $s_k(q)=g_k(q,u)|_{u=u_q}$ where u_q is the solution of the fixed point equation $u=g(q,u)$

$$g(q,u)=\sum_{k\ge1} p(k)\sum_{m\ge1} p_{th}(m|k)\sum_{k_2=0}^{k}\hat{p}(k_2|k)\cdot P[X_{k_2}+Y_{k-k_2}\ge m]+\sum_{k\ge1} p(k)\cdot p_{th}(0|k)$$

$$g_k(q,u)=\sum_{m\ge1} p_{th}(m|k)\sum_{k_2=0}^{k}\hat{p}(k_2|k)\cdot P[X_{k_2}+Y_{k-k_2}\ge m]+p_{th}(0|k)$$

where $X_{k_2}\sim Bin(k_2,\alpha_2 u)$ and $Y_{k-k_2}\sim Bin(k-k_2,\alpha_1 u)$

Proposition

If $\alpha_2>\alpha_1$, then $\frac{\partial s(q)}{\partial q}>0$

Proposition

If $\alpha_2>\alpha_1$, then for all $k\ge1$, $\frac{\partial s_k(q)}{\partial q}\ge0$

Simplified Optimization Problems

$$\min_{0\le\phi\le1} \sum_{k\ge1} c_k\cdot p(k)\cdot\phi(k)\cdot s_k(q)$$

Subject to:

$$q\ge q_\gamma$$

$$q=\frac{1}{\bar{d}}\sum_{k\ge1} k\cdot p(k)\cdot\phi(k)$$

$$\max_{0\le\phi\le1} q \tag{1}$$

Subject to: (2)

$$\sum_{k\ge1} c_k\cdot p(k)\cdot\phi(k)\cdot s_k(q)\le\bar{c} \tag{3}$$

$$q=\frac{1}{\bar{d}}\sum_{k\ge1} k\cdot p(k)\cdot\phi(k) \tag{4}$$

Results

Real World Networks

Table 1: Simple Parameters of the two real-world networks used for simulations.

	Network A	Network B
Source	p2p-Gnutella08	Hamsterster
Network category	Peer-to-peer	Social
Nodes	6301	2426
Edges	20777	16631
Maximum degree	273	97
Average degree	6.59	13.71
Number of triangles	2383	53265
Clustering coefficient	0.01	0.51
Connected components	2	148

Numerical Validation

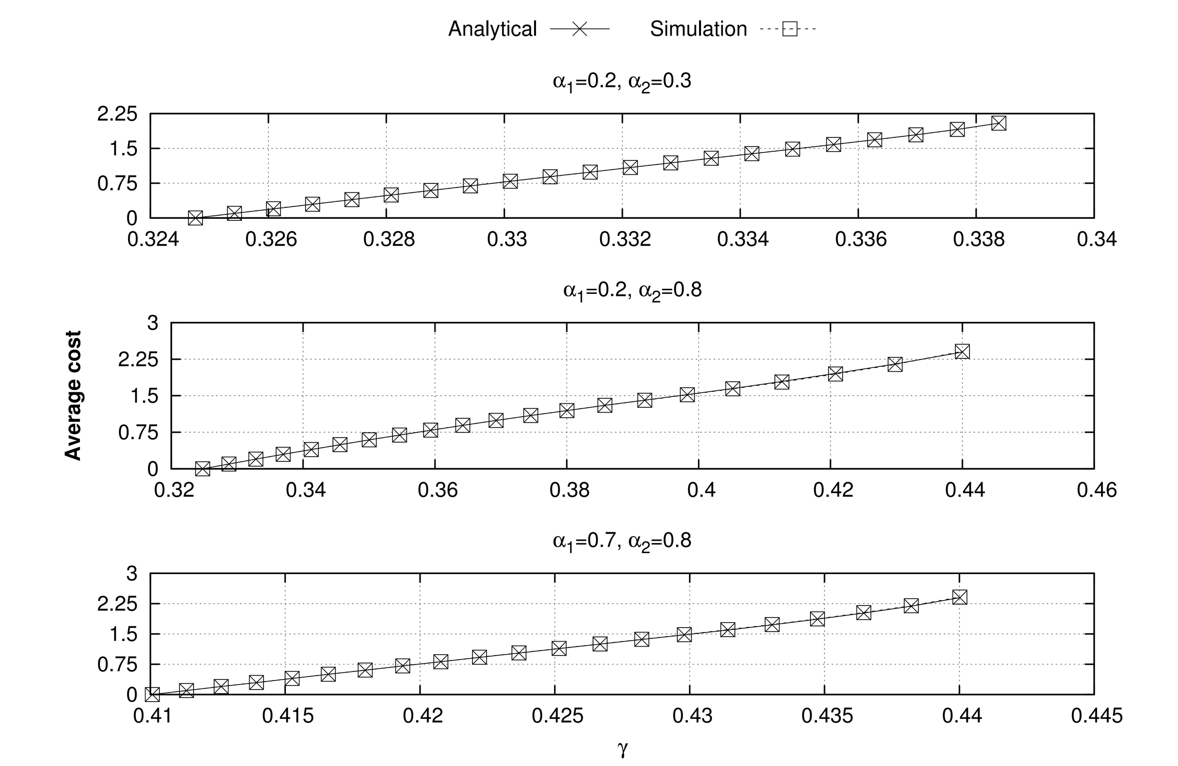

Figure 2: Analytical and simulated values of average cost vs. γ for Gnutella network.

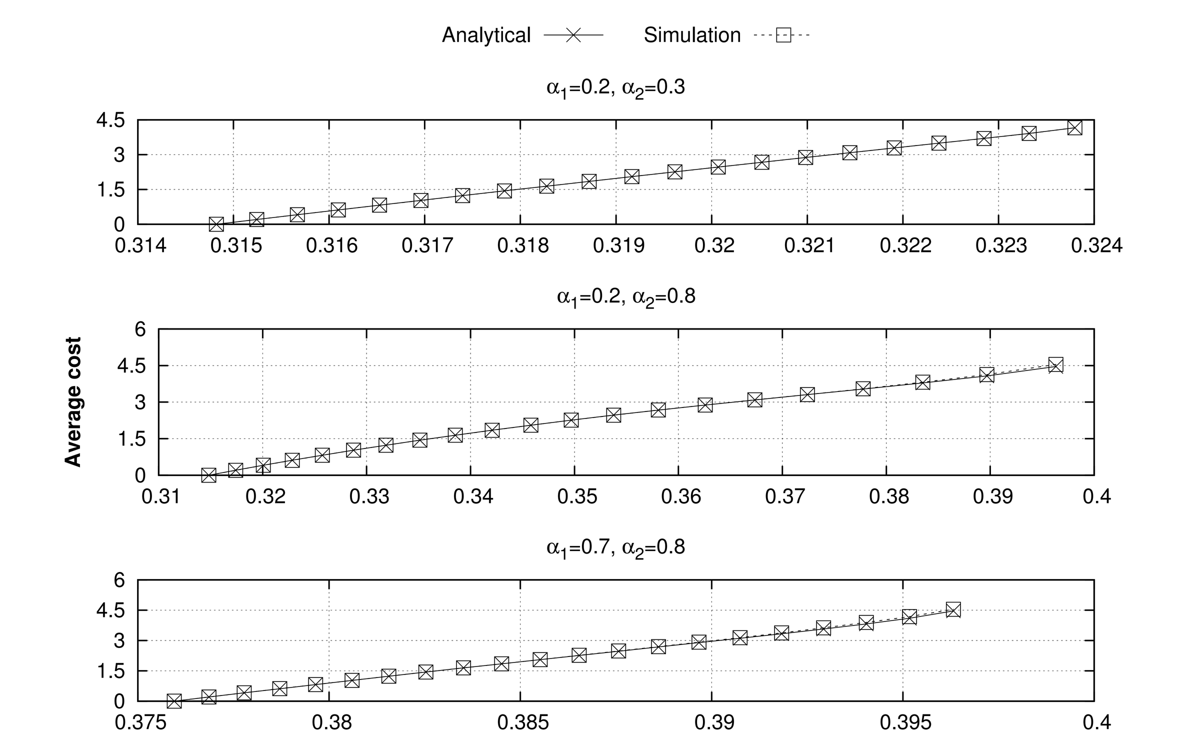

Figure 3: Analytical and simulated values of average cost vs. γ for Hamsterster network.

Conclusion

- ✓ Studied the problem of campaigning in social networks (for marketing a free or freemium service) by offering incentives for referrals.
- ✓ Used results from reliability theory that enabled us to solve the optimization problems with simple algorithms
- ✓ Showed that our analytical results are applicable in real world social networks