An Analysis of RANSAC Algorithm

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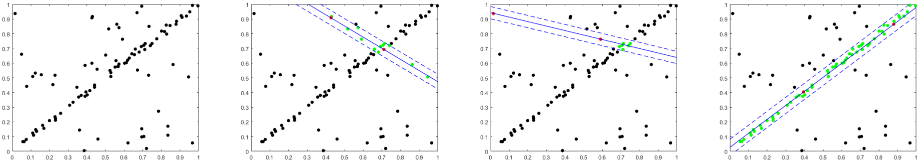
OUTLINE

Introduction to RANSAC

- RANSAC - Random Sampling and Consensus (Fishler and Bolles, 1981)
- An iterative algorithm for robust model parameter estimation.
- At each iteration
  - **Hypothesis step**. Estimate model parameter for a randomly samples subset. Size of the subset is the smallest sufficient for determining parameters. (example - 2 for 2D line, 3 for 3D planes).
  - **Consensus Step**. Collect samples consistent with this model.
- Select hypothesized model to be the true model if consensus set is the largest seen so far.

RANSAC

Data | Iteration-1 | Iteration-2 · · · | Iteration-n



RANSAC

- Simple but powerful.
- Popular among Computer Vision (CV) community, routinely used for
  - Linear Model Fitting (Lines, Planes etc)
  - Image/Point Cloud Registration problems
  - Epipolar geometry and Homography estimation.

Assumptions and Heuristics

- Lower limit on number of iterations K

$$K = \frac{\log(1-p)}{log(1-u^m)}$$

  p - User defined confidence in recovered model
  u - Probability of a randomly selected data point being an inlier
  m - Size of the subset (smallest sufficient)
- Assumption
  - Inliers are noise free, an impractical assumption.
- In practice
  - number of iterations required much higher than K.
  - taking subsets of size greater than minimum gives better results.

Motivation

- We used RANSAC in our work on compression of RGBD (Colour Image + Depth) video data.
- We found that number of iterations required much higher than the value suggested by Fishler and Bolles.
- Taking larger subsets gave us better results.
- Analysis for these observations not available in literature.
- Objective of our Analysis
  - Is there an optimal size of subsets ?
  - Can we tighten the lower bound on K ?

Modification

- Account for error in parameter estimation for inlier only subset.
- Representation for higher than minimal subsets.
- Equation-(1) can be more precisely rewritten as

$$K = \frac{\log(1-p)}{log(1-u^m)} \tag{1}$$

$$K_n = \frac{\log(1-p)}{\log\left(1 - Pr(f_{err}(\theta, \widehat{\theta}) < \epsilon | X_{[n]})\right)} \tag{2}$$

Formulation

- Subsets containing inliers only $X_{[n]}^{in}$
- Subsets containing atleast one outlier $X_{[n]}^{in+out}$

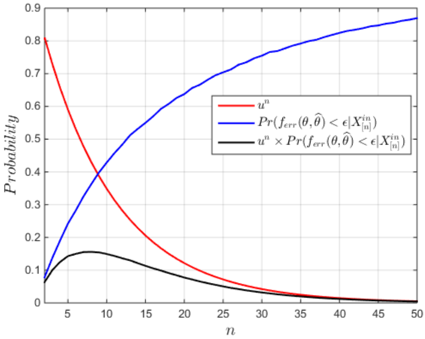
$$
\begin{aligned}
Pr(f_{err}(\theta,\widehat{\theta}) < \epsilon | X_{[n]}) &= \overbrace{u^n \times Pr(f_{err}(\theta,\widehat{\theta}) < \epsilon | X_{[n]}^{in})}^{\text{inliers only subsets}} + \overbrace{(1-u^n) \times Pr(f_{err}(\theta,\widehat{\theta}) < \epsilon | X_{[n]}^{in+out})}^{\text{atleast one outlier}} \\
&= u^n \times Pr(f_{err}(\theta,\widehat{\theta}) < \epsilon | X_{[n]}^{in}) + \Delta_n \qquad (3)
\end{aligned}
$$

- For negligible overlap in distributions of inliers and outliers, $\Delta_n \approx 0$

Formulation

$$Pr(f_{err}(\theta, \widehat{\theta}) < \epsilon | X_{[n]}) \approx u^n \times Pr(f_{err}(\theta, \widehat{\theta}) < \epsilon | X_{[n]}^{in}) \quad (4)$$

- Gaussian perturbation among inliers.
- Least Squares parameter estimator.
- Maximum Likelihood Estimation guarantees asymptotic convergence.
- Expected Behavior

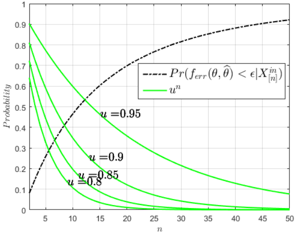


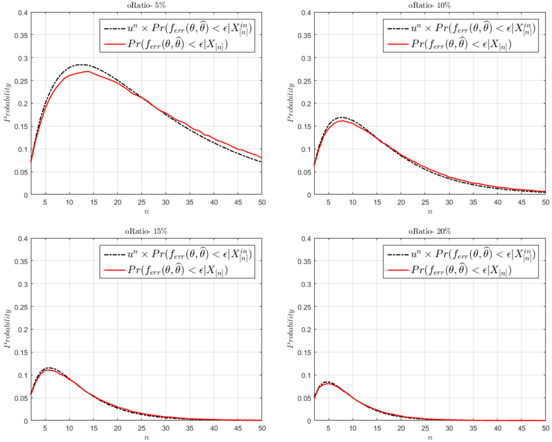
Simulations

- Linear Models

$$W^T X + w_0 = 0, \qquad \text{subject to} \|W\|_2 = 1$$

- 2D-Line parameter estimation



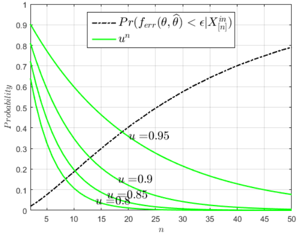


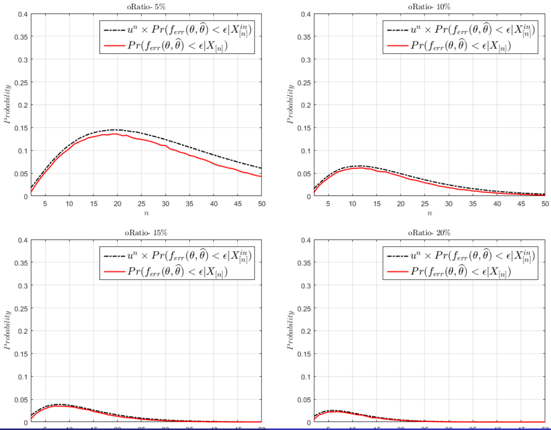
Simulations

- 2D/3D Point Cloud Transformation

$$Y = RX + t \qquad \text{Where, } R \in \mathbb{R}^{d\times d}\text{: rotation}$$
$$t \in \mathbb{R}^{d}\text{: translation}$$

- Simulations for 2D Point Cloud Transformation parameter estimation.





Optimal Subset Size

- Maximum probability of recovering correct model at

$$\begin{aligned} n_{opt} &= \max_n Pr(f_{err}(\theta, \widehat{\theta}) < \epsilon | X_{[n]}) \\ &\approx \max_n u^n \times Pr(f_{err}(\theta, \widehat{\theta}) < \epsilon | X^{in}_{[n]}) \end{aligned} \tag{5}$$

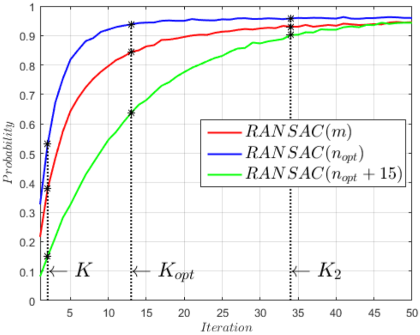
- Lower bound on number of iterations

$$K_{n_{opt}} = \frac{\log(1-p)}{\log(1 - Pr(f_{err}(\theta, \widehat{\theta}) < \epsilon | X_{[n_{opt}]}))} \tag{6}$$

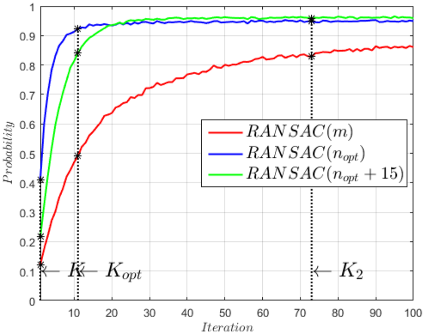
- By definition $Pr(f_{err} < \epsilon | X_{[n_{opt}]}) \geq Pr(f_{err} < \epsilon | X_{[m]}) \quad \Rightarrow K_m \geq K_{n_{opt}}$

Optimal Subset Size and Model Recovery

2D Line



2D Point Cloud



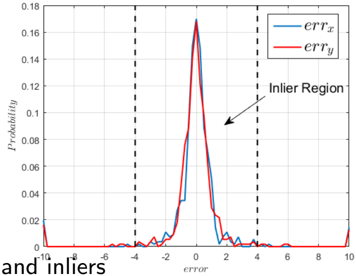
User defined Confidence in model, $p = 0.9$

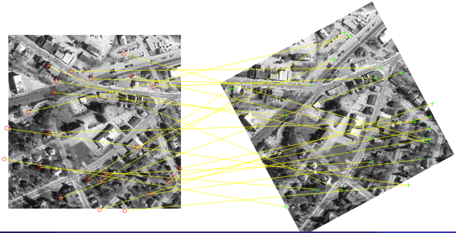
Probability of a random sample is an inlier, $u = 0.9$

Sample Application

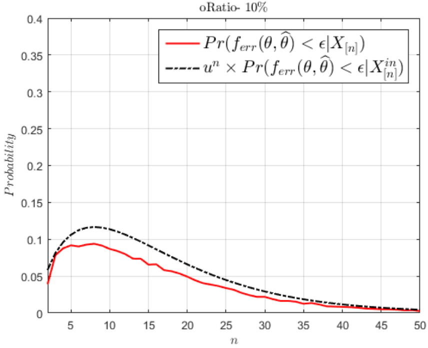
- Recovery of Euclidean Transformation for images (Rotation + Translation)
- Point feature correspondences (SIFT/SURF).
- Errors in point correspondences
- Reasonable Assumptions
    - Inlier noise - Gaussian
    - Negligible overlap in distributions of outliers and inliers

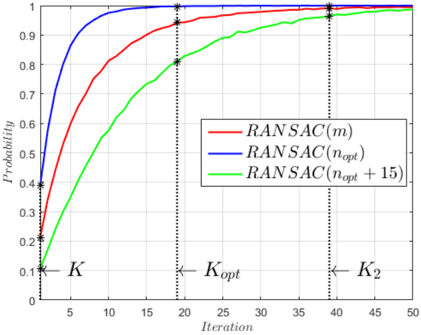
Error Distribution





Comparison with Simulation





Future Work

- Estimation of Homography and Epipolar Geometry.
- Comparison with various variants of RANSAC.

Summary

- When is it better to take subsets of size greater than minimum.
- Optimal size of the subsets.
- Tighter Lower bound on the number of iterations of RANSAC.

References

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Thank You