Stable Galerkin Finite Element Formulation for the Simulation of Electromagnetic Flowmeter

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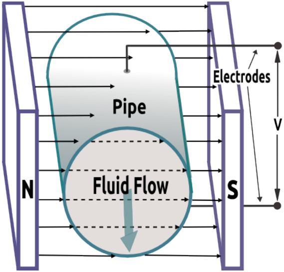
Electromagnetic flowmeter



- Electromagnetic flowmeter is extensively employed for the measurement of liquid-metal flow rate in fast breeder reactors.
- Reliable measurement is essential for the control and safe operation of the reactor

- Experimental calibration is extremely difficult
- Theoretical approach is preferred

$$\sigma\nabla\phi \;-\; (\nabla\cdot\frac{1}{\mu}\nabla)\mathbf{A} - \sigma\ \mathbf{u}\times\nabla\times\mathbf{A} = \sigma\ \mathbf{u}\times\mathbf{B_a}$$

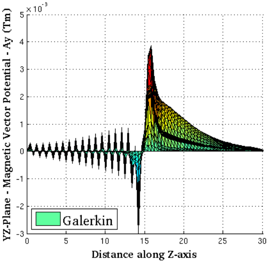
$$\nabla\cdot(\sigma\nabla\phi) - \nabla\cdot(\sigma\ \mathbf{u}\times\nabla\times\mathbf{A}) = \nabla\cdot(\sigma\ \mathbf{u}\times\mathbf{B_a})$$

- Galerkin finite element method (GFEM) is a ready choice
- Only in very limited literature whole 3D version of the problem is simulated using GFEM [6]

Numerical simulation of Electromagnetic Flowmeter

- GFEM is known to suffer from numerical oscillations when $Pe = \mu\sigma|\mathbf{u}|\Delta z/2 > 1$. As remedy Streamline upwind/Petrov Galerkin (SU/PG) scheme is suggested in the allied literature [1] [2].

- SU/PG scheme introduces boundary error [4] [5] and non-physical current in the solution [8]
- In addition, SU/PG scheme needs calculation of stabilization parameter and requires more calculation for higher order elements.



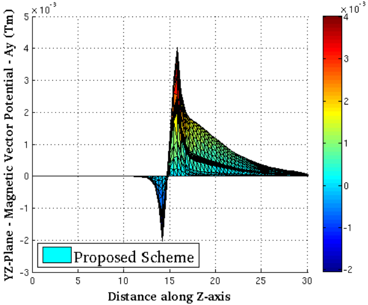
- Scope of the work: To arrive at a 'Stable Galerkin Finite Element Formulation for Electromagnetic Flowmeter Analysis'

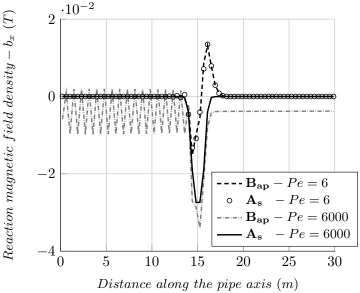
Approach

- Classically, numerical stability of the FEM solution is analyzed with the 1D version of the problem [3] [9]
- FEM equations for a regular grid takes the form of difference equation, which is employed for the required analysis
- In this work, the Z-transform approach is proposed so as to bring tools from control systems theory
- Accordingly for GFEM., relation between vector potential of reaction magnetic field (A_y) and the input field (B_x) can be written as, $\frac{A_y}{B_x} \simeq \frac{\Delta z}{3} \; \frac{(Z+0.27)(Z+3.73)}{(Z-1)\,{\color{red}(Z+1)}}$
- Pole at '-1' is responsible for the numerical oscillations
- Proposed approach: To seek re-formulation of the RHS so as to introduce necessary zeros
- **Scheme-1:** Input field on the RHS is restated in terms of magnetic vector potential [7]: $\frac{A_y}{A_{sy}} \simeq -\frac{(Z-1)\,{\color{red}(Z+1)}}{(Z-1)\,{\color{red}(Z+1)}}$

Proposed Scheme -1 - Simulation Results for flowmeter

- 33598 brick elements with graded structured mesh in flow direction ($\mu = 4\pi \times 10^{-7} Hm^{-1}$, $\sigma_{sodium} = 7.21 \times 10^{6}\ Sm^{-1}$, $\sigma_{steel} = 1.16 \times 10^{6}\ Sm^{-1}$)

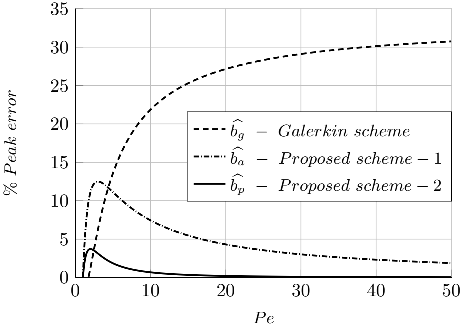




- Works well only when input field varies only in the flow direction

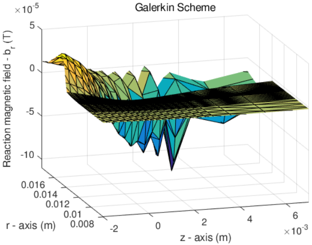
Proposed Scheme - 2

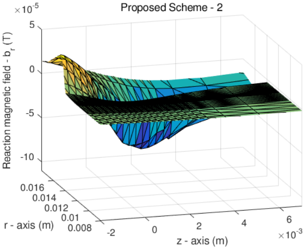
- Weighted nodal input magnetic field is considered where the required weights are constrained so as to be consistent, as well as, brings in necessary zero
- **Scheme 2:** $\frac{A_y}{B_x} \simeq \frac{\Delta z}{2} \quad \frac{(Z+1)^2}{(Z\ -\ 1)\,(Z+1)}$

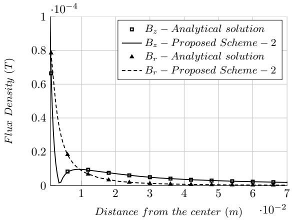


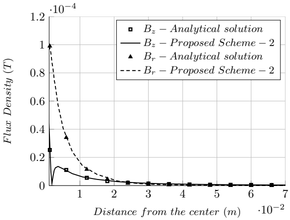
- Performs better than 'scheme−1' - double zeros at '-1'.
- For both the schemes, extensive 1D and 2D Z-transform analysis has been performed to ascertain the characteristics of the numerical solution

Application to other moving conductor problems









- Scheme-2 gives stable results and it is matching well with the analytical solution of the TEAM-9 standard test problem

Summary and Conclusion

- Theoretical evaluation of the sensitivity of electromagnetic flowmeter is a preferred choice for liquid metal flow measurement
- Only numerical approach is feasible and GFEM is a ready choice. The GFEM suffer from numerical instability, when $Pe > 1$.
- Existing remedial measures in allied fields like SU/PG scheme gives non-physical solutions at the boundary.
- Two novel stable schemes have been proposed for graded regular mesh along the flow direction.
- Accurate results have been obtained for flowmeter and similar problems even at very high flow rates/velocity

Thank you

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