Incentivized Campaigning in Social Networks

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Topics in Thesis

- Stochastic Analysis of Epidemics in Adaptive Time Varying Networks.
- Percolation in Dependent and Antagonistic Interacting Networks.
- Cost Effective Campaigning in Social Networks.
- Incentivized Campaigning in Social Networks.

Publications

- B. Kotnis, and J. Kuri. "Stochastic analysis of epidemics on adaptive time varying networks." *Physical Review E 87.6 (2013): 062810.*
- B. Kotnis, and J. Kuri. "Percolation on networks with antagonistic and dependent interactions." *Physical Review E 91.3 (2015): 032805.*
- B. Kotnis, and J. Kuri. "Cost Effective Campaigning in Social Networks." *Elsevier Physica A:Statistical Mechanics and its Applications 450 (2016):670.*
- B. Kotnis, A. Sunny, and J. Kuri. "Incentivized Campaigning in Social Networks." Manuscript under review at *IEEE/ACM Transactions on Networking.*
- A. Sunny, B. Kotnis, and J. Kuri (2015). "Dynamics of history-dependent epidemics in temporal networks." *Physical Review E*

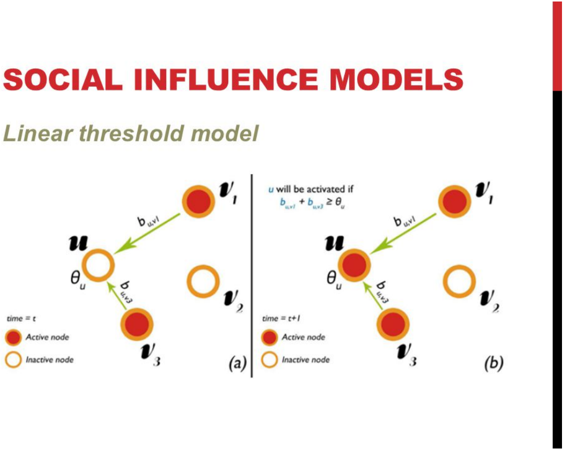
Motivation

- Social recommendation of products and services by providing incentives (viral marketing) is very useful for enlarging the market share.
- Uber, Lyft, Living Social provide incentives to recommend services and products to friends.
- Campaign : Flipkart wants people to adopt and buy using their smart phone app.
- Anyone who persuades friends to install the app gets a discount coupon.
- Providing discounts to everyone may be too costly. Many (like Dropbox) put a threshold on the incentives.

The Problem

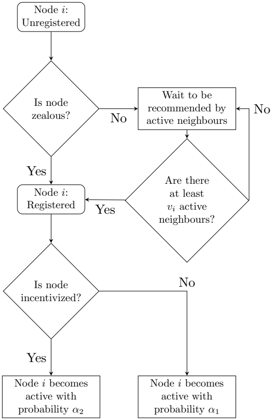
- Goal : Maximize the number of individuals that download the Flipkart app, or maximize app downloads (campaign size) for a given cost budget.
- Don't know the structure of the social network. Have an estimate of number of friends of a randomly chosen individual (degree distribution).
- After an individual installs the app, the app can find out the *exact number* of friends (not their identities).
- Incentives increase the chance that an app user refer the app.
- Incentives provided after they install the app. The higher the referrals more the discount (higher cost).

Campaign Model



- Seeds: Core Flipkart buyers that have downloaded the app. Call them Zealous nodes (randomly chosen).

Model



- Assume an uncorrelated and locally treelike network.

Optimization Problem

- Compute individuals to be incentivized that minimize average cost for achieving a given campaign size γ
- $s(q)$: proportion of nodes that have installed the app. $s_k(q)$: nodes with k neighbors that have installed the app. $\phi(k)$: probability that k degree node has been incentivized. $p(k)$: proportion of nodes with degree k.

$$\min_{\mathbf{0}\leq\phi\leq\mathbf{1}} \quad \sum_{k\geq 1} c_k \cdot p(k) \cdot \phi(k) \cdot s_k(q)$$

Subject to:

$$s(q) \geq \gamma$$

$$q = \frac{1}{d} \sum_{k\geq 1} k \cdot p(k) \cdot \phi(k)$$

Size of the Campaign : Math is complicated

- $s(q) = g(q,u)|_{u=u_q}$ and $s_k(q) = g_k(q,u)|_{u=u_q}$ where u_q is the solution of the fixed point equation $u = g(q,u)$

$$g(q,u) = \sum_{k\geq 1} p(k) \sum_{m\geq 1} p_{th}(m|k) \sum_{k_2=0}^{k} \hat{p}(k_2|k) \cdot P[X_{k_2} + Y_{k-k_2} \geq m] + \sum_{k\geq 1} p(k) \cdot p_{th}(0|k)$$

$$g_k(q,u) = \sum_{m\geq 1} p_{th}(m|k) \sum_{k_2=0}^{k} \hat{p}(k_2|k) \cdot P[X_{k_2} + Y_{k-k_2} \geq m] + p_{th}(0|k)$$

where $X_{k_2} \sim Bin(k_2, \alpha_2 u)$ and $Y_{k-k_2} \sim Bin(k - k_2, \alpha_1 u)$

Main Result

- Intuition: Campaign size is an increasing function of proportion of incentivized nodes.

- α_2 chance of referring if node has been incentivized. α_1 chance of referring if node has not been incentivized.

Theorem

If $\alpha_2 > \alpha_1$, *then* $\frac{\partial s(q)}{\partial q} > 0$

Theorem

If $\alpha_2 > \alpha_1$, *then for all* $k \geq 1$, $\frac{\partial s_k(q)}{\partial q} \geq 0$

Simplified Optimization Problem

$$\min_{\mathbf{0} \leq \boldsymbol{\phi} \leq \mathbf{1}} \quad \sum_{k \geq 1} c_k \cdot p(k) \cdot \phi(k) \cdot s_k(q)$$

Subject to:

$$q \geq q_\gamma$$

$$q = \frac{1}{d} \sum_{k \geq 1} k \cdot p(k) \cdot \phi(k)$$

$$\max_{\mathbf{0} \leq \boldsymbol{\phi} \leq \mathbf{1}} \quad q$$

Subject to:

$$\sum_{k \geq 1} c_k \cdot p(k) \cdot \phi(k) \cdot s_k(q) \leq \overline{c}$$

$$q = \frac{1}{d} \sum_{k \geq 1} k \cdot p(k) \cdot \phi(k)$$

Real World Networks

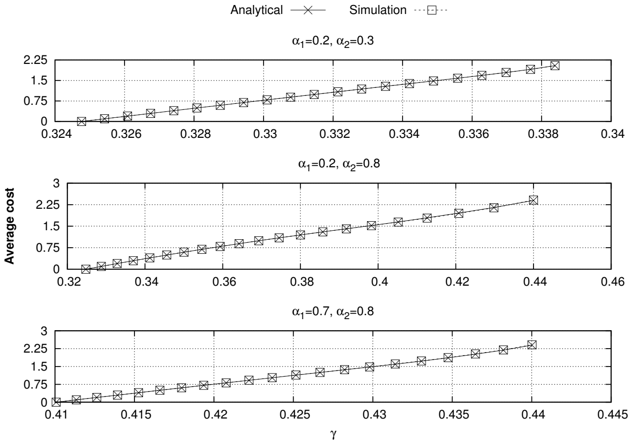


Figure: Analytical and simulated values of average cost vs. γ for Gnutella network.

Conclusions

- Studied the problem of campaigning in social networks (for marketing a free or freemium service) by offering incentives for referrals.
- Used percolation theory to compute the campaign size and formulate optimization problems.
- Used results from reliability theory that enabled us to solve the optimization problems with simple algorithms
- Showed that analytical results are applicable in real world social networks