Temporal Self-Organization: A Reaction-diffusion Framework for Spatio-temporal Memories

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Problems solved thus far

- Understand and model the memory formation using Turing's reaction-diffusion equations.
- Explain the recall/anticipation process.
- Reconstruction of missing samples using Turing's reaction-diffusion equations.

Idea and the architecture

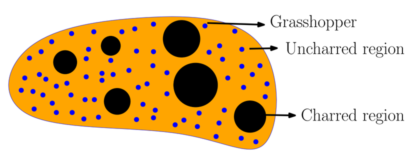

Figure 1: Example.

Figure 2: Cheetah spots.

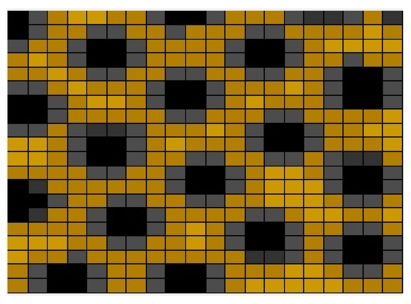

Figure 3: Reconstruction.

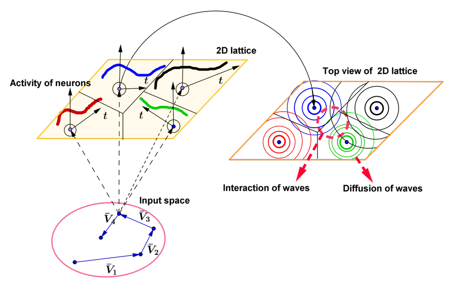

Figure 4: Architecture.

- Absence of Grasshoppers: No pattern.
- Slow Grasshoppers: No pattern.
- Fire $\longleftrightarrow$ Grasshoppers: reaction part.
- Hopping of Grasshoppers: diffusion part.

Reaction-diffusion equation

- The general form reaction-diffusion equation is given by

$$\frac{\partial A}{\partial t} = F(A,B) + D_A\nabla^2 A$$
$$\frac{\partial B}{\partial t} = G(A,B) + D_B\nabla^2 B$$

- $A,\ B$ are concentrations of the morphogens.
- G and H are, in general, non-linear functions of A and B.
- D_A and D_B are the diffusion coefficients of A and B respectively.

Competitive-Cooperative model

- Winner: based on spatio-temporal metric.
- Winner: activator; losers: inhibitors.
- Interconnected neurons: reaction part.
- Propagation of electrical pulse: diffusion part.

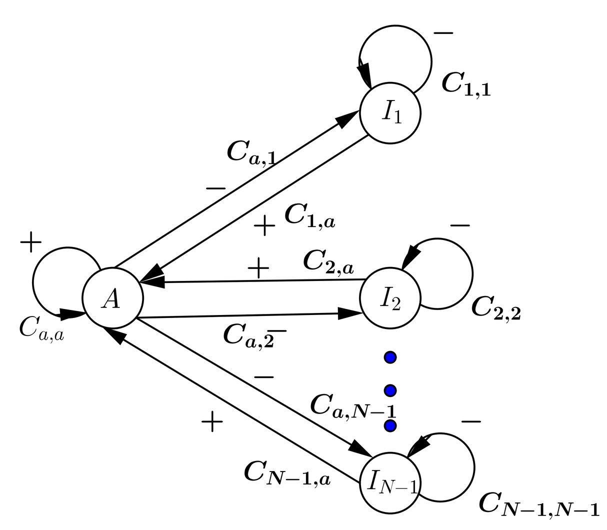

Figure 5: Competitive-cooperative model for neuronal interactions.

Sampling of wave solution

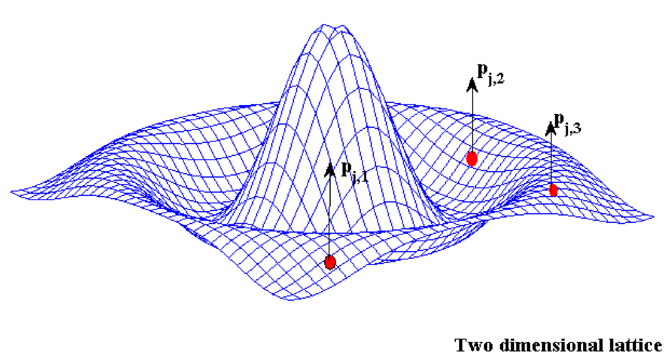

Figure 6: Sampling of wave solution.

- The wave solution is sampled at the positions $\overline{W}_j$ to get P_{ji}.

Spatio-temporal metric and the learning rule

- Spatio-temporal metric is given by

$$\Delta_j = \left(\left\|\overline{V} - \overline{W}_j\right\| + 1 - \alpha_j\right).$$

- Potential function is given by

$$E = \frac{1}{2}\sum_{j=1}^{N}\int_{\mathbb{R}^d} \Delta_j^2 h_j P\left(\overline{V}\right) \mathrm{d}\overline{V}^d.$$

- E is convex.
- E is minimized using stochastic gradient descent method.

Simulation experiments and results

- Goal: Illustrate the working of our algorithm by demonstrating the spatio-temporal structure in the input data embedded within the activations of the neurons.

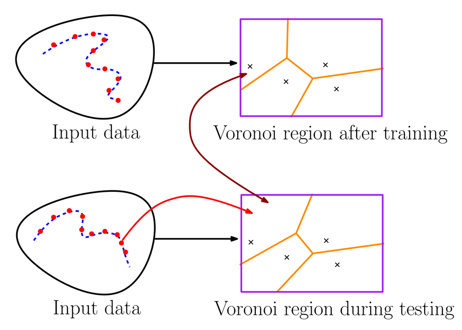

Figure 7: Illustration of anticipation capability of our algorithm.

Data Generation: Lorenz dynamical system

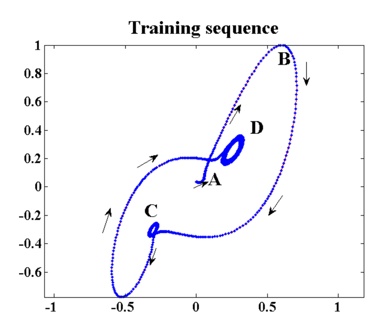

Figure 8: Training seq.

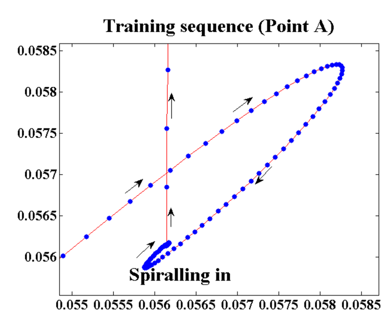

Figure 9: Section A.

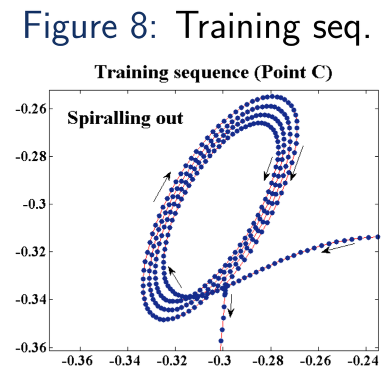

Figure 10: Section C.

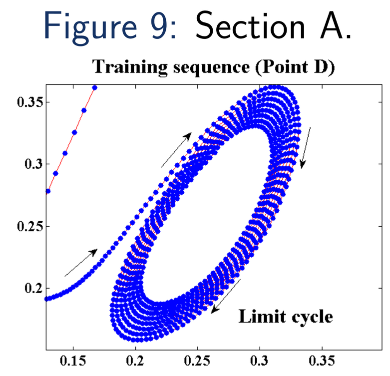

Figure 11: Section D.

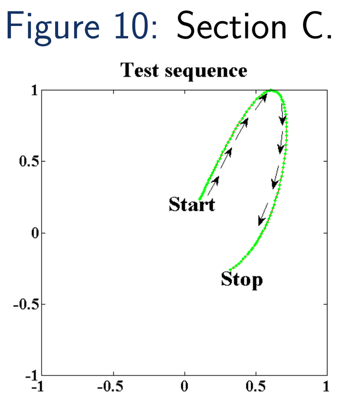

Figure 12: Test seq. 1.

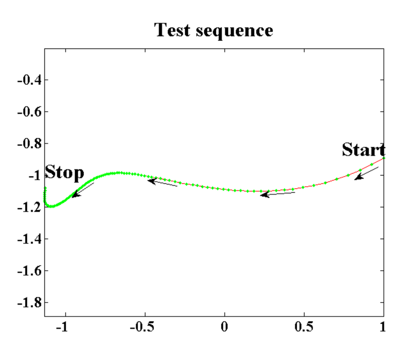

Figure 13: Test seq. 2.

Simulation results with test sequence-1

- Expansion of Voronoi region. Observe the Voronoi regions $9 \to 2 \to 2 \to 1$.

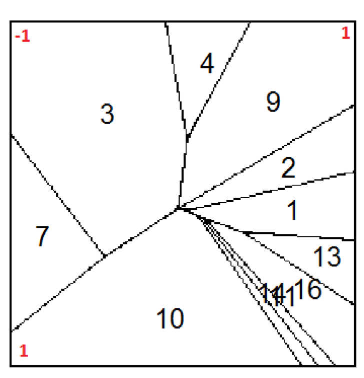

Figure 14: 9 expands.

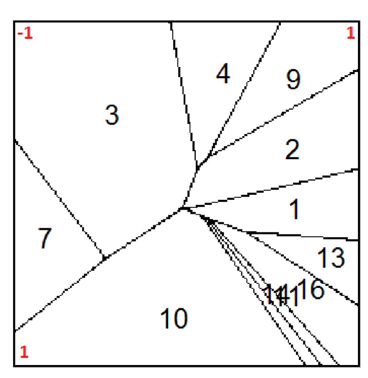

Figure 15: 2 expands.

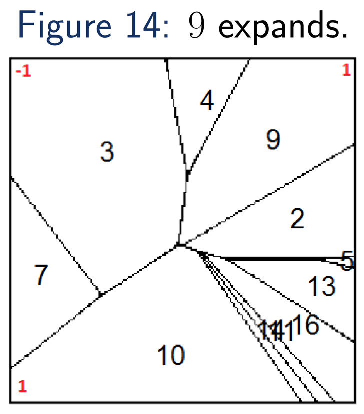

Figure 16: 2 expands.

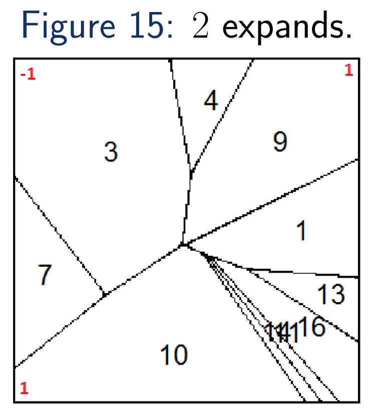

Figure 17: 1 expands.

Simulation results with test sequence-2

- Expansion of Voronoi region. Observe the Voronoi regions $9 \to 3 \to 3 \to 3$.

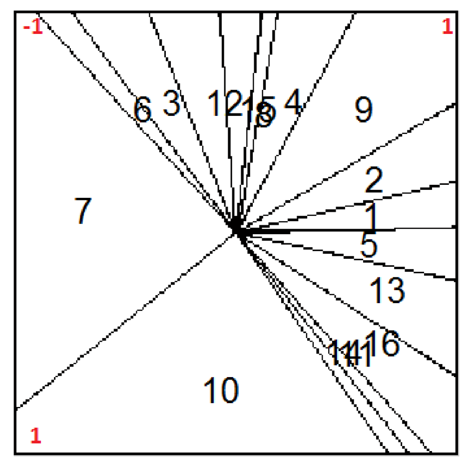

Figure 18: 9 expands.

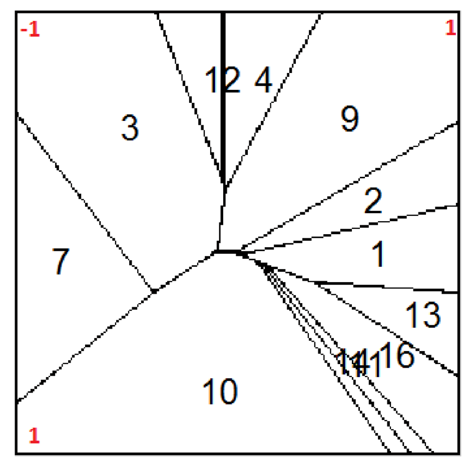

Figure 19: 3 expands.

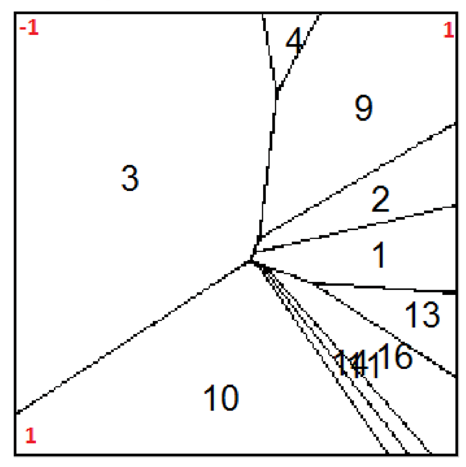

Figure 20: 3 expands.

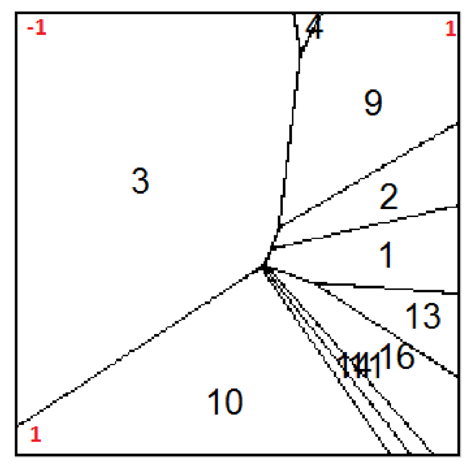

Figure 21: 3 expands.

Logistic equation and comparison results

- Data is generated using Logistic equation given by

$$\overline{V}_{n+1} = a(\overline{V}_n - \overline{V}_n^2),$$

- Correlated noise sequence n_c is generated as follows:

$$n_c(n) = 0.2n_c(n-1) + 0.2n_c(n-2) + 0.2n_c(n-3) + 0.2n_c(n-4) + 0.3n_c(n-5).$$

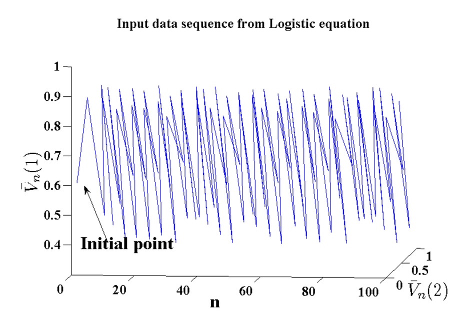

Figure 22: Clean data.

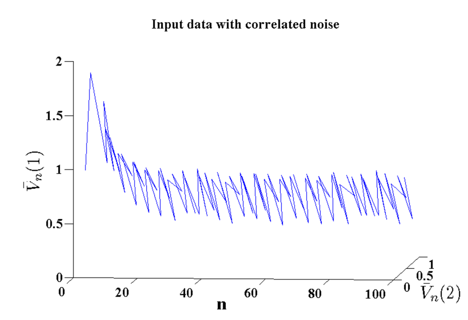

Figure 23: Noisy data.

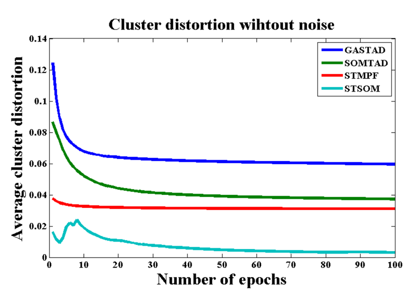

Figure 24: No noise.

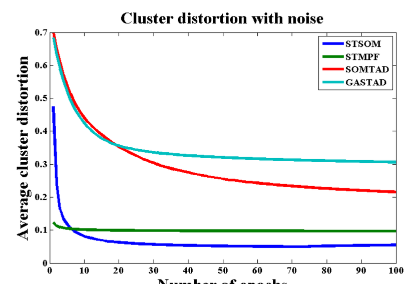

Figure 25: With noise.

- STSOM performs better under noiseless and noisy conditions compared to the SOMTAD, GASTAD, and the STMPF algorithms.
- A glitch in the beginning of the STSOM performance curve: every neuron has the same probability of winning the competition for an input data in the beginning.

References

[1] P. Gowgi and S. Garani, 'Temporal Self Organization: A Reaction diffusion Framework for Spatio-temporal Memories', under review in *IEEE Transactions on Neural Networks and Learning Systems*, 2016.

[2] P. Gowgi and S. Garani, 'Density Transformation and Parameter Estimation from Back Propagation Algorithm', in *IEEE Intl. Joint Conf. on Neural Netw.*, IJCNN, 2016.

[3] P. Gowgi and S. Garani, 'Spatio-temporal map formation based on a Potential Function', in *IEEE Intl. Joint Conf. on Neural Netw.*, IJCNN, 2015.

[4] P. Gowgi and S. Garani, 'Missing Sample Reconstruction by Learning Spatio-temporal difference Sequence', *to be submitted*.

[5] J.D. Murray, "Mathematical Biology II," *Springer-Verlag New York*, vol. 2, 2003.

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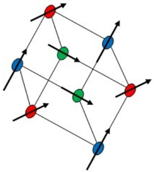
April 07, 2017



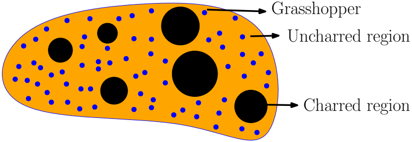
Overview

- Solved problems.
- From cheetah spots to spatio-temporal memories.
- Architecture and analysis.
- Simulation setup and results.

Problems solved thus far

- Understand and model the memory formation using Turing's reaction-diffusion equations.
- Explain the recall/anticipation process.
- Reconstruction of missing samples using spatio-temporal memory.

Idea

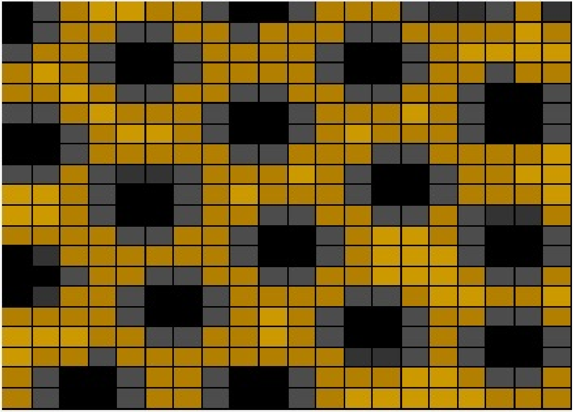


- Absence of Grasshoppers: No pattern.
- Slow Grasshoppers: No pattern.
- Fire $\longleftrightarrow$ Grasshoppers: reaction part.
- Hopping of Grasshoppers: diffusion part.

$$\frac{\partial A}{\partial t} = F(A, B) + D_A \nabla^2 A$$
$$\frac{\partial B}{\partial t} = G(A, B) + D_B \nabla^2 B$$

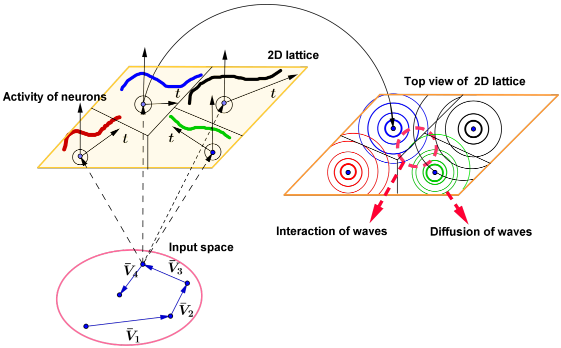
J.D. Murray, "Mathematical Biology II," *Springer-Verlag New York*, vol. 2, 2003.

Cheetah spots: simulated



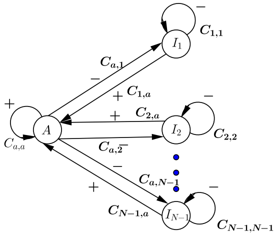
Reconstructed cheetah spots using Turing's reaction-diffusion equations.

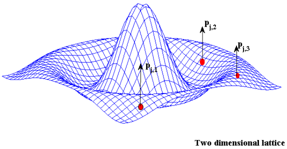
Architecture



- Winner: based on spatio-temporal metric.
- Winner: activator; losers: inhibitors.
- Interconnected neurons: reaction part.
- Propagation of electrical pulse: diffusion part.

Competitive-cooperative model





Sampling of wave solution.

Theorem

To ensure positivity of activity waves

$$C_{1,1} > 2\,|C_{a,1}| + \sum_{j\neq a}^{N-1} |C_{a,j}| - C_{a,a}, \tag{1}$$

$$C_{j,j} > |C_{a,j}| + |C_{a,j-1}| + C_{j-1,j-1} \tag{2}$$

for all $j = 1, \cdots, N-1.$

Spatio-temporal metric

- Spatio-temporal metric is given by

$$\Delta_j = \Big(\|\overline{V} - \overline{W}_j\| + 1 - \alpha_j \Big). \tag{3}$$

Lemma (Distance metric)

$d(\overline{V}, \overline{W}_i) = \Big(\|\overline{V} - \overline{W}_i\| + 1 - \alpha_i \Big)$ *is a distance metric.*

- Potential function is given by

$$E = \frac{1}{2} \sum_{j=1}^{N} \int_{\mathbb{R}^d} \Delta_j^2 h_j P(\overline{V}) \mathrm{d}\overline{V}^d. \tag{4}$$

Lemma (Distance metric)

The potential function $E = \frac{1}{2} \sum_{j=1}^{N} \Delta_j^2 h_j$ *is convex.*

Learning rule

- The learning rule is given by

$$\overline{W}_i(n+1) = \overline{W}_i(n) - \eta(n)\frac{\partial E}{\partial \overline{W}_i}, \quad (5)$$

Theorem (Convergence)

Let E be a real valued continuous function which is at least twice differentiable, i.e., $E \in \mathcal{C}^2$ on Ω. Then, E converges to $E^* \leq E(\Omega)$ if $\eta(n) \geq \frac{(\sigma-1)\nabla E^{\mathrm{T}}(n) d^{(n)}}{L\left\|d^{(n)}\right\|^2}$. Let $\nabla E \in \mathbb{R}^d$ be the gradient of E, $d^{(n)} \in \mathbb{R}^d$ such that $\nabla E^{\mathrm{T}} d^{(n)} < 0$ with $\sigma \in [\rho, 1]$ for some $\rho > 0$.

Simulation experiments and results: Lorenz dynamical system

- Goal: Illustrate the working of our algorithm by demonstrating the spatio-temporal structure in the input data embedded within the activations of the neurons.

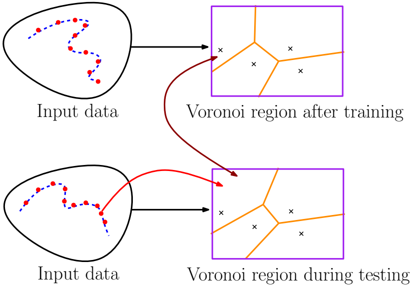
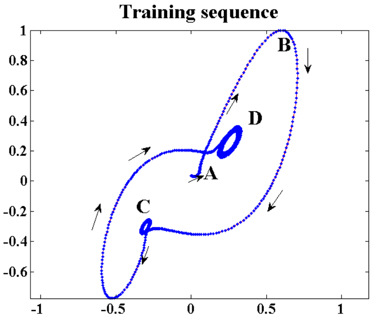
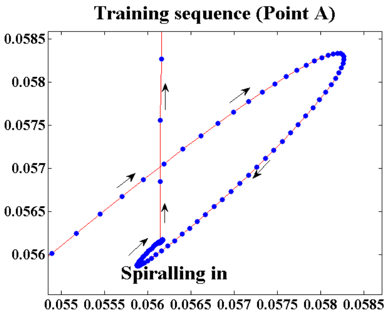


Illustration of anticipation capability of our algorithm.

Data generation

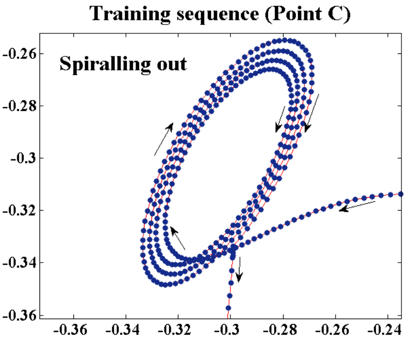


Training sequence with all the modes.

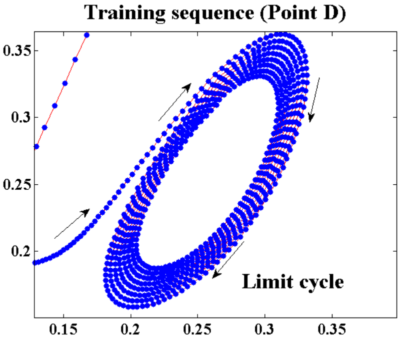


Section A of the training sequence.

Data generation

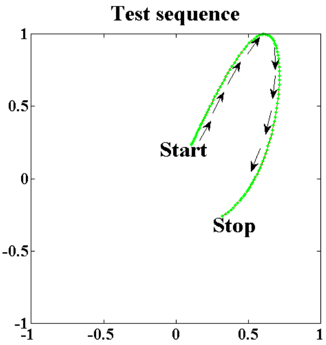


Section C of the training sequence.

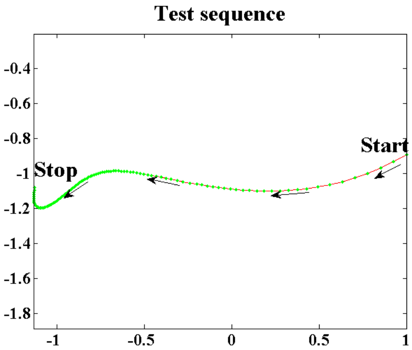


Section D of the training sequence.

Data generation

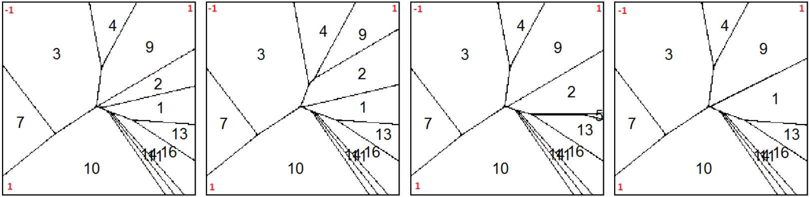


Test sequence 1.



Test sequence 2.

Dynamic Voronoi tessellations: test sequence 1

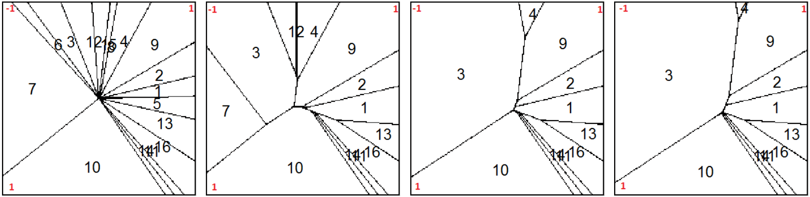


- Observe: $9 \rightarrow 2 \rightarrow 2 \rightarrow 1$.

Theorem

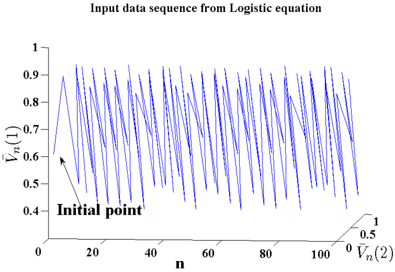
The expansion or the contraction of a Voronoi region is local and volume conserving.

Dynamic Voronoi tessellations: test sequence 2



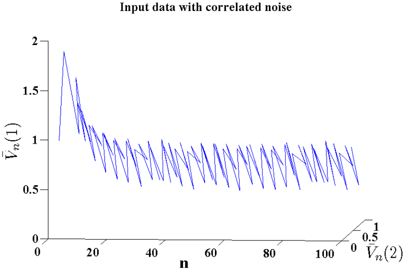
- Observe: $9 \rightarrow 3 \rightarrow 3 \rightarrow 3$.

Experiment 2: Logistic equation



Data generated from Logistic equation.

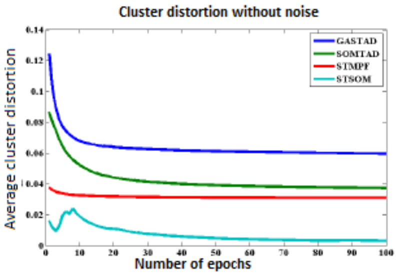
$$\overline{V}_{n+1} = a(\overline{V}_n - \overline{V}_n^2).$$

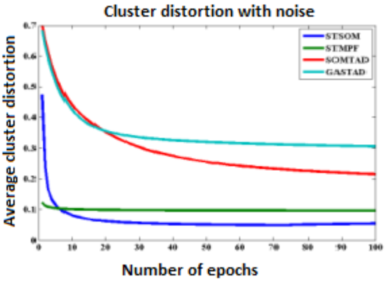


Data with correlated noise.

$$\begin{aligned} n_c(n) &= 0.2n_c(n-1) + 0.2n_c(n-2) \\ &+ 0.2n_c(n-3) + 0.2n_c(n-4) \\ &+ 0.3n_c(n-5). \end{aligned}$$

Performance comparison





Cluster distortion without noise

Cluster distortion with noise

- STSOM: Spatio-temporal self-organizing maps.[1]
- STMPF: Spatio-temporal map formation based on potential function.[2]
- SOMTAD: Self-organizing maps with temporal activity diffusion.[3]
- GASTAD: Neural gas with temporal activity diffusion.[3]

Reference

[1] P. Gowgi and S. Garani, "Temporal Self Organization: A Reaction diffusion Framework for Spatio-temporal Memories", under review in *IEEE Transactions on Neural Networks and Learning Systems*, 2016.

[2] P. Gowgi and S. Garani, "Spatio-temporal map formation based on a Potential Function", in *IEEE Intl. Joint Conf. on Neural Netw.*, IJCNN, 2015.

[3] S. Garani and J. Principe, "Dynamic Vector Quantization Of Speech", in *Advances in Self-Organising Maps.*, Springer London, 2001.

[4] P. Gowgi and S. Garani, "Density Transformation and Parameter Estimation from Back Propagation Algorithm", in *IEEE Intl. Joint Conf. on Neural Netw.*, IJCNN, 2016.

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Thank you