Packing and Covering with Geometric Objects

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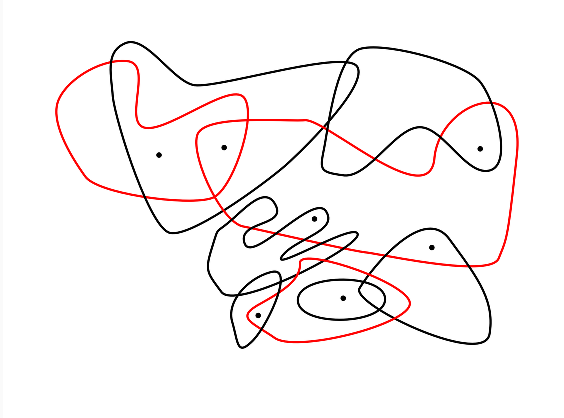
Abstract

We consider Geometric **Packing** and **Covering** problems that are NP-hard and design efficient approximation algorithms to solve them. We show that these problems admit Polynomial Time Approximation Schemes (PTAS) using **Local Search** algorithms.

Set Cover Problem

Given a set of regions $\mathcal{R}$ and a point set P, compute the minimum sized subset $\mathcal{R}' \subseteq \mathcal{R}$ such that every point in P is contained in at least one object in $\mathcal{R}'$.



Set Packing Problem

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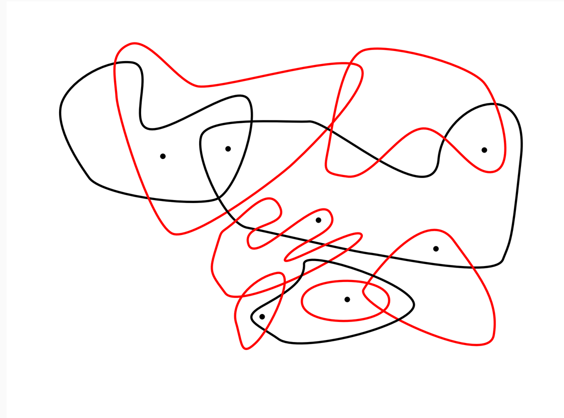


Local Search Algorithm

Input: $\mathcal{R}$, P, ϵ (error parameter).

- Start with some feasible solution.
- Change the current solution by making local changes (spending $n^{O(1/\epsilon^2)}$ time) if it improves the objective function.
- Return the current solution if local changes can no longer improve the solution.

Output: $(1+\epsilon)$-approximate solution in $n^{O(1/\epsilon^2)}$ time.

Analysis of Approximation factor

The Local Search algorithm yields an approximation factor of $(1+\epsilon)$ if the following properties hold.

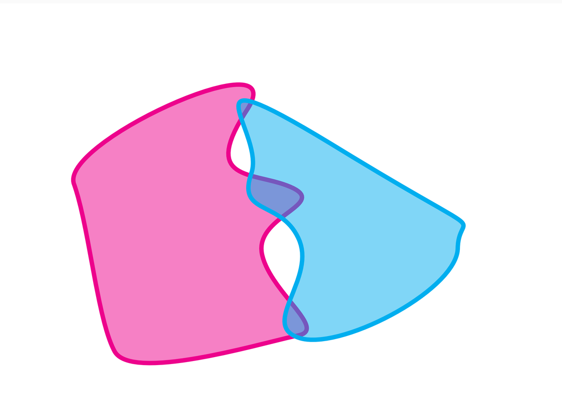
- A balanced vertex separator of sublinear size, e.g., Planar graphs.
- Small Set Expansion, i.e., Every small subset (size $O(1/\epsilon^2)$) of the LARGER SET expands in the smaller set.

Our Problems

1. Shallow Packing
2. Point Packing
3. Runaway Rectangle Escape problem
4. Unique Coverage
5. Multi-Covering problem
6. Prize Collecting Set Cover
7. Art Gallery problems

Our Geometric Objects

The Geometric Objects we have mostly studied are set of Non-Piercing Regions. A set of objects $\mathcal{R}$ are said to be non-piercing if for every $A, B \in \mathcal{R}$ the following holds.



- $A \setminus B$ and $B \setminus A$ are connected regions.
- The boundaries of A and B intersect at most k times, i.e., $|\partial A \cap \partial B| \leq k$ where k is some constant.

Shallow Packing

Definition. Given $\mathcal{R}$ and P and an integer capacity, at most some constant, for every point in P, compute the maximum sized subset $\mathcal{R}' \subseteq \mathcal{R}$ such that every point in P is contained in most as many objects in $\mathcal{R}'$ as its capacity.

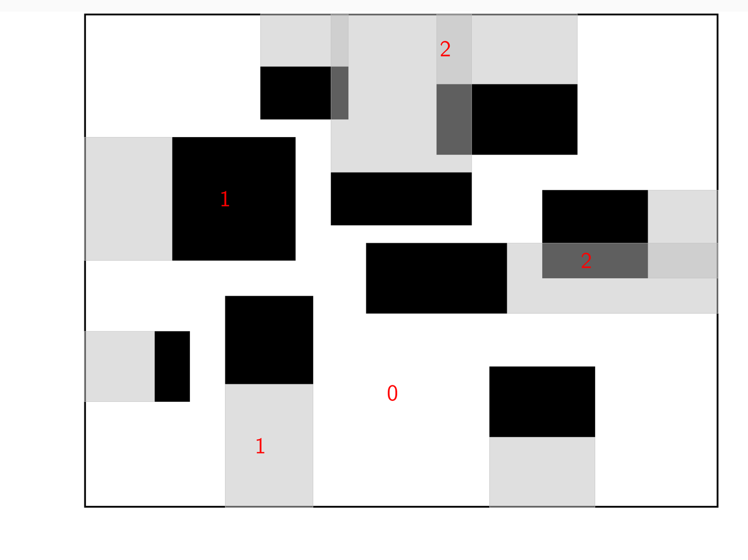
Our Contribution. We show the existence of appropriate graphs that have balanced separators of sublinear size which are not planar. We further extend the result for even broader class of objects when they have **sub-quadratic union complexity**.

Intersection Graphs of Shallow Arrangements

- Given $\mathcal{R}$ and P such that the depth of every point in P is at most some constant ℓ, we define a graph G over $\mathcal{R}$ and put an edge between R_i and R_j if $R_i \cap R_j \cap P \neq \emptyset$.
- Observe that for $\ell \geq 5$, G need not be planar. As, K_5 can possibly exist as a subgraph of G.
- We prove that still the graph G has a balanced separator of sublinear size using some appropriate planar graphs. This proof works for non-piercing regions and in the continuous setting (when $P = \mathbb{R}^2$) it works for set of objects with sub-quadratic union complexity.

Runaway Rectangle Escape problem

Definition. Given a set of rectangles $\mathcal{R}$ and the maximum allowed density (number of layers) d, maximize the number of rectangles that can escape in one of the 4 directions.



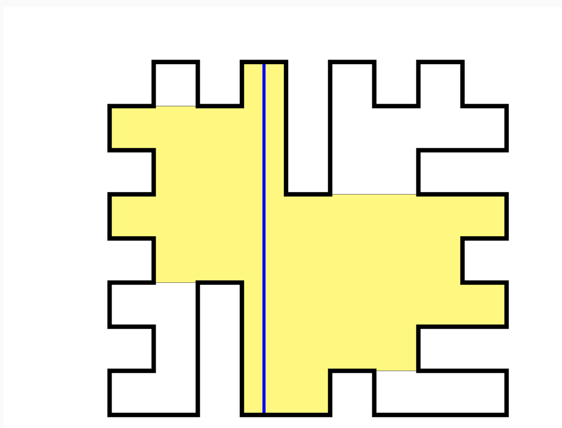
Our Contribution. We give a $(2+\epsilon)$-approximation algorithm.

Covering Problems

- **Unique Coverage.** Given $\mathcal{R}$ and P compute a subset $\mathcal{R}' \subseteq \mathcal{R}$ such that the number of points in P that are contained by exactly one object from $\mathcal{R}'$ is maximized.
- **Multi-Covering Problem.** Given $\mathcal{R}$, P and an integer demand of every point in P, compute the minimum sized subset $\mathcal{R}' \subseteq \mathcal{R}$ such that every point in P is contained in at least as many objects from $\mathcal{R}'$ as it demand.
- **Prize Collecting Set Cover.** Given $\mathcal{R}$, P, a weight for every $R \in \mathcal{R}$ and a penalty for every point $p \in P$, compute a subset $\mathcal{R}' \subseteq \mathcal{R}$ such that the sum of the weights of objects in $\mathcal{R}'$ plus the sum of the penalties for the points not covered by $\mathcal{R}'$ is minimized.

Our Contribution. We give an $(1+\epsilon)$-approximation algorithm with sparsity assumptions on the input.

Art Gallery Problems



We consider a variant of the art gallery problem where the art gallery is orthogonal in shape and sliding cameras (along orthogonal axis) are planned to be installed to guard the gallery. In the adjacent figure, the floor plan of an art gallery is shown which is guarded by a single vertical sliding camera (shown in blue). The region in yellow is its visibility area and thus there are places that remain unguarded. The objective is to compute the minimum number of sliding cameras such that the entire region is guarded.

Publication

1. The Runaway Rectangle Escape Problem (with S. Govindarajan, A. Maheshwari, N. Misra, S. C. Nandy, S. Shetty) CCCG '14, arXiv 1603.04210
2. Packing and Covering with Non-Piercing Regions (with S. Govindarajan, R. Raman, S. Ray) ESA '16
3. Local Search strikes again: PTAS for variants of Geometric Covering and Packing (with P. Ashok, S. Govindarajan) under review
4. Effectiveness of Local Search for Art Gallery and Prize Collecting Problems (with S. Bandyapadhyay) under review

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Geometric Optimization
Problems that are
Computationally Hard

Approximation Algorithms
that run in
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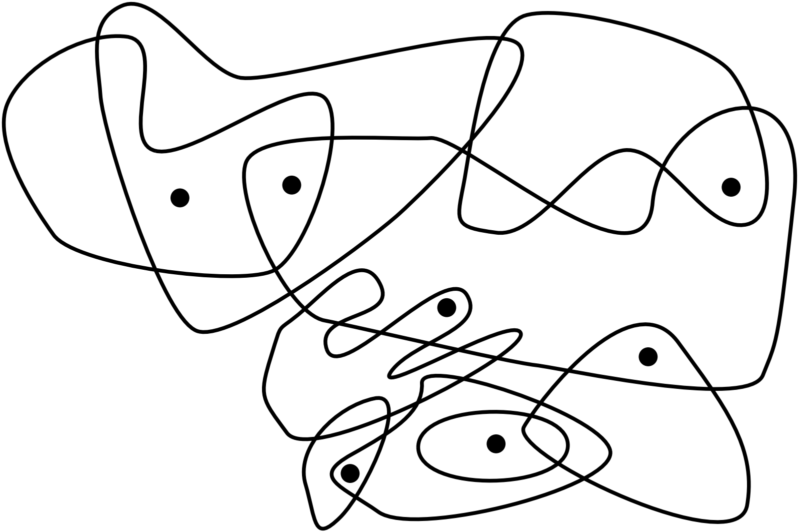
Packing and Covering Problems that are NP-Hard

Approximation Algorithms that run in Polynomial Time

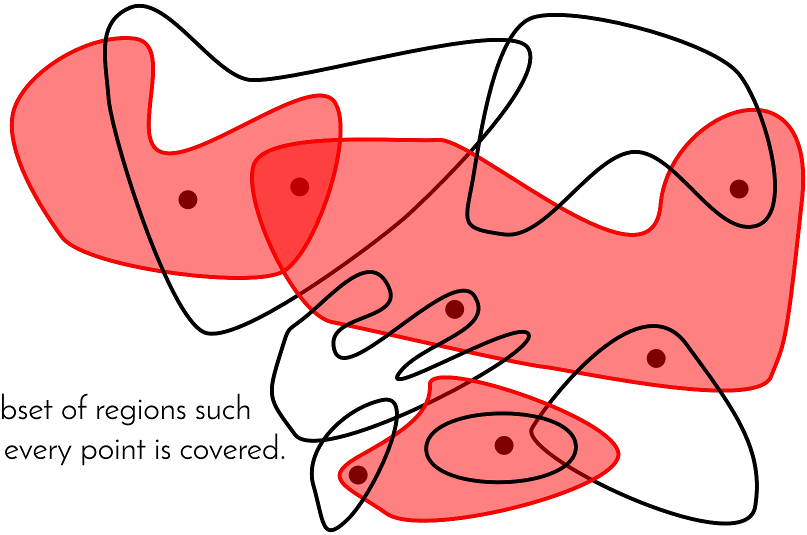
Packing and Covering Problems that are NP-Hard

Local Search Algorithms that run in Polynomial Time

Packing and Covering Problems that are NP-Hard

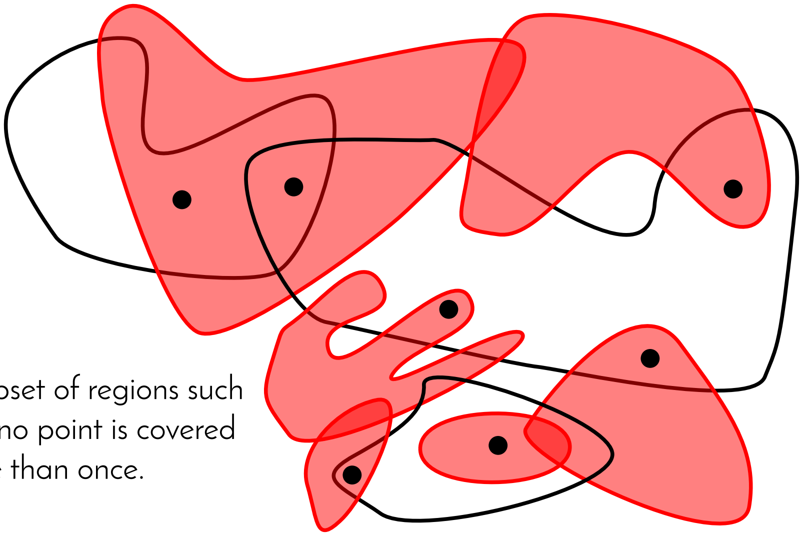


Set Cover



A subset of regions such that every point is covered.

Set Packing



A subset of regions such that no point is covered more than once.

Packing and Covering Problems that are NP-Hard

Local Search Algorithms that run in Polynomial Time

Shallow Packing

Point Packing

Runaway Rectangle Escape problem

Unique Coverage

Multi-Covering problem

Prize Collecting Set Cover

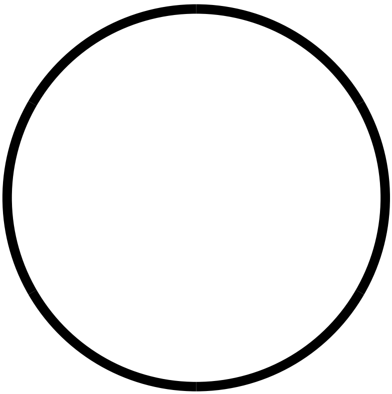
Art Gallery problems

Local Search Algorithms that run in Polynomial Time

Local Search Algorithms that run in Polynomial Time

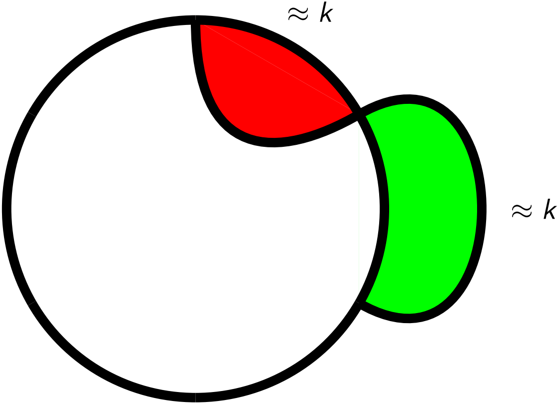
Local Search

parameter k



Local Search

parameter k



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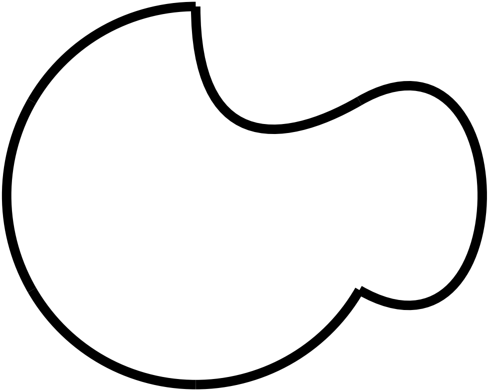

Local Search

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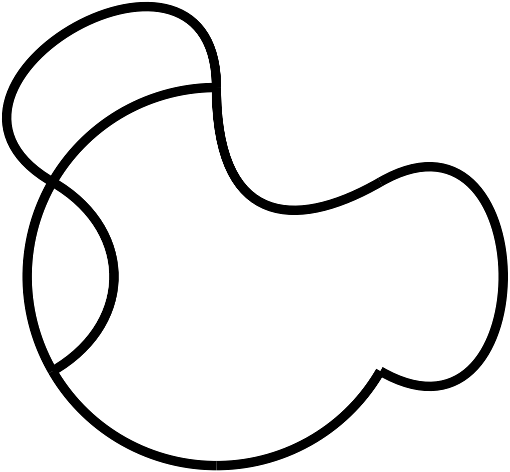

Local Search

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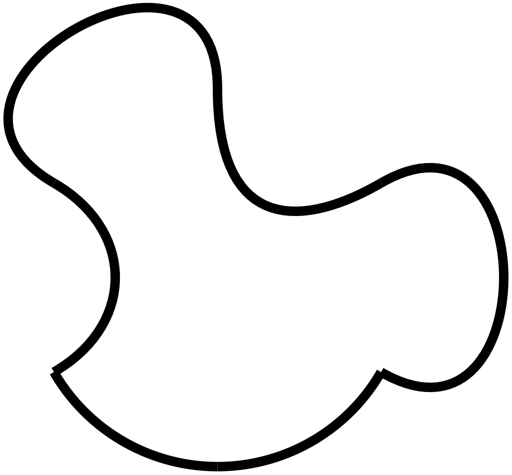

Local Search

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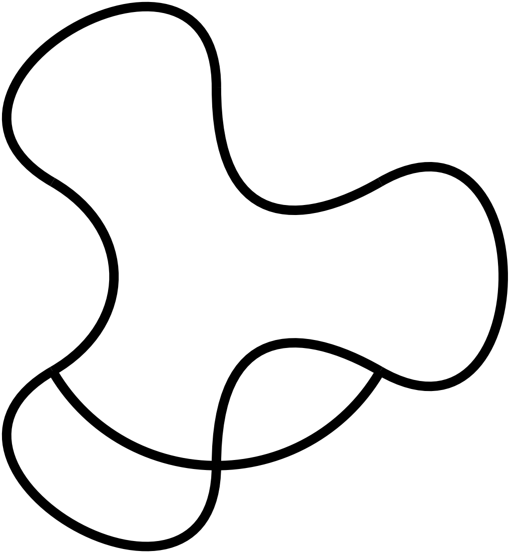

Local Search

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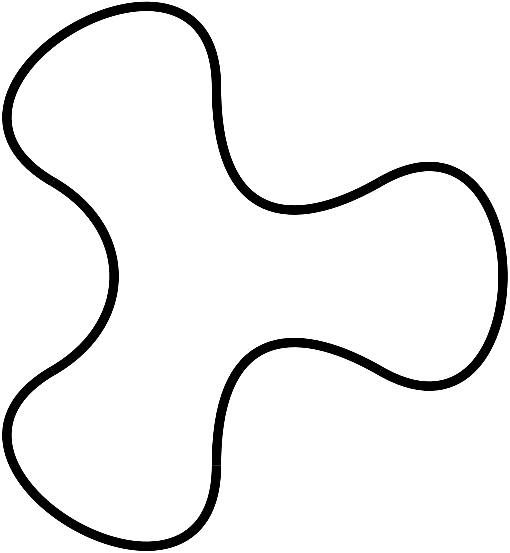
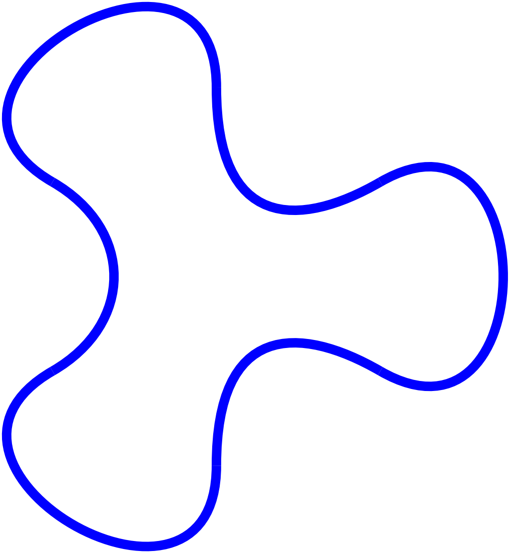


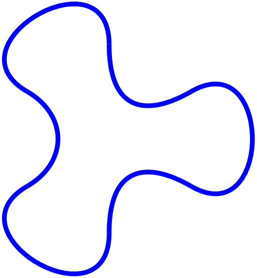
Local Search Solution



$(1+\epsilon)$−approximation

$\epsilon = c/\sqrt{k}$

Local Search Solution



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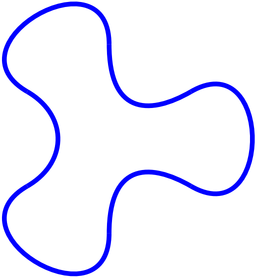
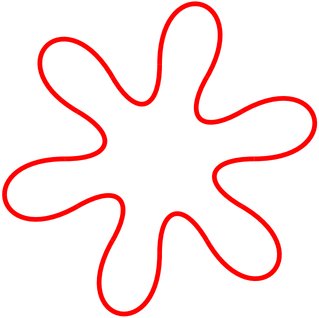


Optimum Solution



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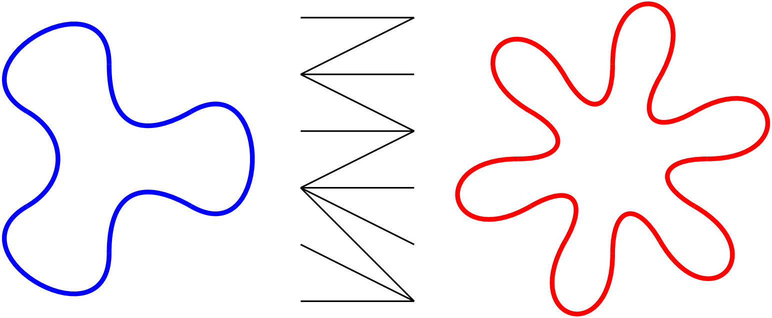
Local Search Solution

Optimum Solution



Bipartite Graphs with Small and Balanced Separators viz., **Planar Graphs**

$(1+\epsilon)-$approximation

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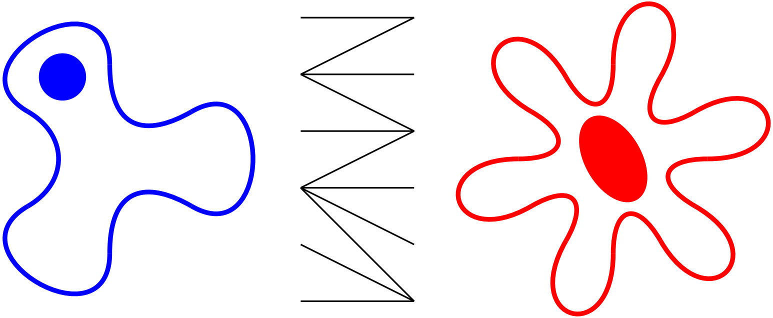
Local Search Solution

Optimum Solution



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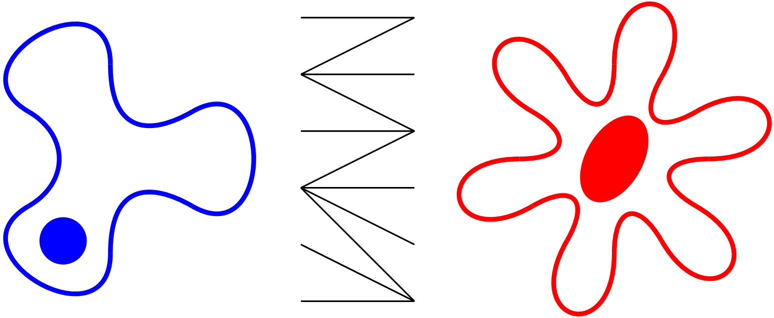

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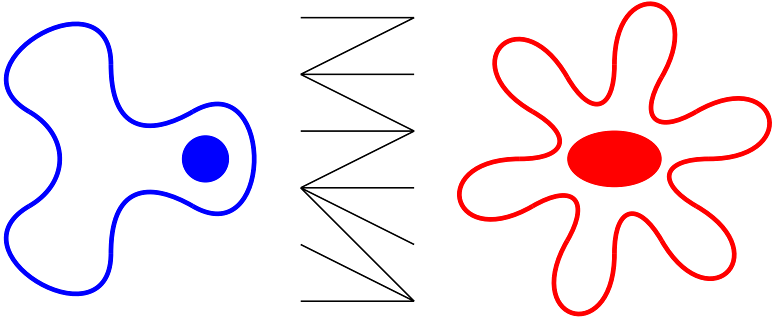

Bipartite Graphs with Small and Balanced Separators viz., **Planar Graphs**

Small Set Expansion

Intersection Graphs
of shallow arrangements

NOT Planar

Intersection Graphs
of shallow arrangements

have Small and
Balanced Separators
using some appropriate
planar graphs

Shallow Packing[2,3]

Point Packing[2]

Runaway Rectangle Escape problem[1]

Unique Coverage[3]

Multi-Covering problem[3]

Prize Collecting Set Cover[4]

Art Gallery problems[4]

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Thank You