Differential Encoding for Real-Time Status Updates

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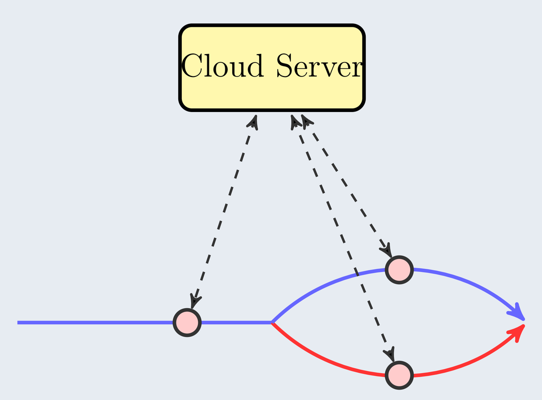
Motivation



- Critical to know the status update before decision making

System Model

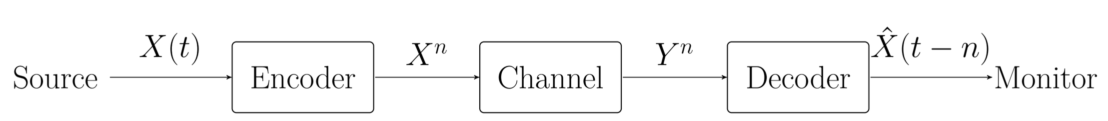


Figure 1: We show an abstract discrete time communication model for a source with m-bit information $X(t)$ at time t.

- **Source**:- We assume that the source message can be represented by finitely many bits, say m and the difference $X(t)-X(t-n)$ can be represented by $1 \le k \le m$ bits.

Problem Statement

How to encode message at the temporally correlated source for timely update? Should one send the current state or the difference between the current and the past state?

- **Encoder**:- The encoded message corresponding to the true state $X(t)$ and the difference $X(t)-X(t-n)$ are called *true update* and *incremental update* respectively. Each message after encoding is of n bits.
- **Channel**:- We consider a bit-wise *iid* binary symmetric erasure channel and each bit is erased with probability ϵ.
- **Decoder**:- Probability of decoding failure for true and incremental updates are given by $p_1 = \mathbb{E}P(n, n-m, E)$ and $p_2 = \mathbb{E}P(n, n-k, E)$.

Performance Metric

Last successfully decoded source state at time t was generated at $U(t)$. Information age [1] $A(t)$ at time t is given by

$$A(t) = t - U(t).$$

We are interested in limiting value of average age

$$\lim_{t\to\infty} \frac{1}{t} \sum_{s=1}^{t} A(s).$$

Update Transmission Schemes

1. **True Updates**:- Each opportunity send *true update*.
2. **Incremental Updates without Feedback**:- Periodically send the *true update* after q updates. In between true updates, send *incremental updates*.
3. **Incremental Updates with Feedback**:- Send the *true update* after each decoding failure. In between true updates, send *incremental updates*.

Renewal Reward Theorem

Let S_i denote the time instant of the ith successful reception of the true update. For all three schemes, the ith inter-renewal time $T_i = S_i - S_{i-1}$ is *iid*. Accumulated age in ith renewal period is given by

$$S(T_i) = \sum_{t=S_{i-1}}^{S_i - 1} A(t)$$

is also *iid*. By renewal reward theorem, the limiting average age is

$$\mathbb{E}A \triangleq \lim_{t\to\infty} \frac{1}{t} \sum_{s=1}^{t} A(s) = \mathbb{E}S(T_i)/\mathbb{E}T_i.$$

True Update

- Inter-renewal time $T_i = nZ_i$
- Number of true update in ith renewal interval Z_i
- $\{Z_i : i \in \mathbb{N}\}$ is *iid* geometric with $(1-p_1)$

Theorem

Limiting average age of for the true update scheme is a.s.

$$\mathbb{E}A \triangleq \lim_{t\to\infty} \frac{1}{t} \sum_{s=1}^{t} A(s) = (n-1)/2 + n/(1-p_1).$$

Incremental Updates without Feedback

- Inter-renewal time $T_i = nqZ_i$
- Number of successfully decoded contiguous incremental updates $\bar{W}_i - 1$ in the ith renewal interval
- $\bar{W}_i$ is the number of successfully decoded updates in ith renewal interval

Theorem

Limiting average age for the incremental updates without feedback is

$$\mathbb{E}\bar{A} \triangleq \lim_{t\to\infty} \frac{1}{t} \sum_{s=1}^{t} \bar{A}(s) = \frac{\mathbb{E}T_i^2}{2\mathbb{E}T_i} + \frac{n^2 \mathbb{E}\bar{W}_i(\bar{W}_i - 1)}{2\mathbb{E}T_i} - \left(n\mathbb{E}(\bar{W}_i - 2) + \frac{1}{2}\right).$$

Incremental Updates with Feedback

- Inter-renewal time $T_i = nZ_i + nW_i$
- Number of incremental updates W_i in ith renewal interval
- $\{W_i : i \in \mathbb{N}\}$ are *iid* geometric with success parameter p_2

Theorem

Limiting average age for the incremental updates with feedback is

$$\mathbb{E}\hat{A} \triangleq \lim_{t\to\infty} \frac{1}{t} \sum_{s=1}^{t} \hat{A}(s) = \frac{(3n-1)}{2} + \frac{n(\mathbb{E}Z_i^2 + \mathbb{E}Z_i)}{2(\mathbb{E}W_i + \mathbb{E}Z_i)}.$$

Analytical Comparison

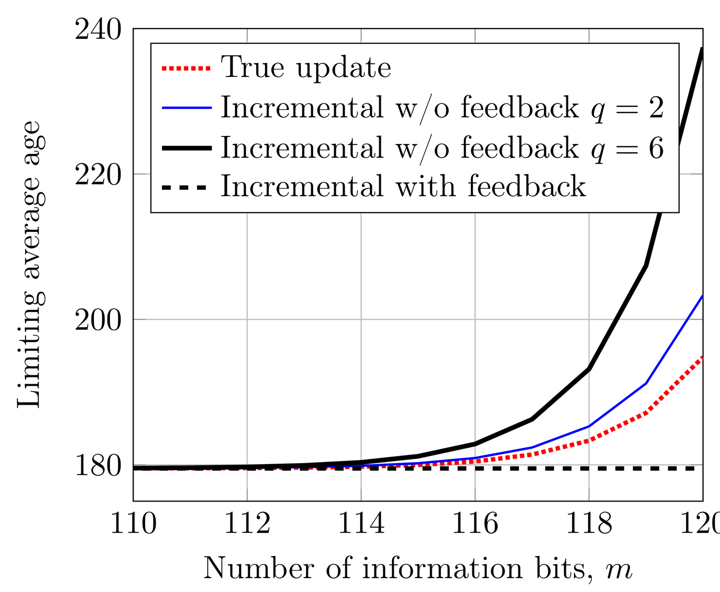
Theorem

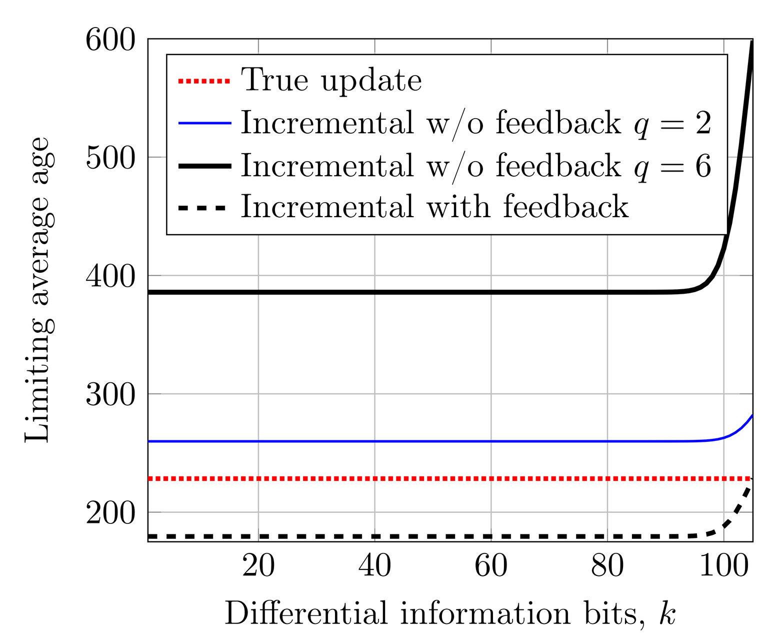
The mean age for the three schemes satisfy [2],

$$\mathbb{E}\hat{A} \le \mathbb{E}A \le \mathbb{E}\bar{A}.$$

Numerical Comparision

For illustration purpose we use a random coding scheme. The code length is fixed to $n = 120$ and the number of information bits is set to $m = 105$.





References

[1] Sanjit Kaul, Roy Yates, and Marco Gruteser. Real-time status: How often should one update? In *IEEE INFOCOM*, pages 2731–2735, 2012.

[2] Sanidhay Bhambay, Sudheer Poojary, and Parimal Parag. Differential encoding for real-time status updates. In *IEEE Wireless Communications and Networking Conference*, March 2017.