

**Syllabus for M.Tech. (AI) Written Test.**

1. **Programming and Data Structures:** Programming in C. Recursion. Arrays, stacks, queues, linked lists, trees, binary search trees, binary heaps, graphs.
2. **Calculus:** Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Theorems of integral calculus, evaluations of definite and improper integrals. Taylor series. Partial derivatives, multiple integrals, vector identities, directional derivatives.
3. **Linear Algebra and Matrix Analysis:** Vector space, basis, linear dependence and independence, matrix algebra, rank, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.
4. **Probability and Statistics:** Random variables. Uniform, normal, exponential, Poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem. Correlation and regression analysis.
5. **Discrete Mathematics:** Propositional and first order logic, sets, relations, functions, partial orders and lattices, groups, graphs: connectivity, matching, coloring. Combinatorics: counting, recurrence relations, generating functions.

**Sample Written Test:** Please see the next few pages for a sample written test. Please note that no model answers will be provided.

Name: \_\_\_\_\_ Application Number: \_\_\_\_\_

**Instructions.** You will have 60 minutes to answer the questions. This paper has fifteen questions. The first set of five questions are on Programming in C and Data Structures; each of these questions carries 2 points. The remaining questions are on Calculus, Linear Algebra and Matrix Analysis, Probability and Statistics, and Discrete Mathematics and carry one point each. For multiple choice questions, you must tick all the correct choices to get credit.

1. (2 points) Give example values of the variables  $x$ ,  $y$ ,  $i$ , and  $j$  that can produce the value TRUE for the following expression:

$$(x > y) \ \&\& \ (i > 0) \ || \ (j < 5)$$

Write your answer here: \_\_\_\_\_

2. (2 points) Give example values of the variables  $x$  and  $xmin$  so that the following program segment will output  $-5$ .

```
{
    if (abs(x) < xmin) x = (x>0)? xmin : -xmin;
    printf ("%d", x);
}
```

Write your answer here: \_\_\_\_\_

3. (2 points) What is the mathematical function computed by the following C-function?

```
float eval (int x) /* assume x >= 0 */
{int i ; float temp = 4.2, value = 1;
  if (x == 0) return 1 ;
  else for ( i = 0 ; i < x ; i++)
      value *= temp ;
  return value ;
}
```

Write your answer here: \_\_\_\_\_

4. (2 points) What is the number of times the printf statement is executed in the following program?

```
for ( i = 1 ; i < 5 ; i++ )
    for ( j=i+1; j<6; j++)
        printf ("%d,%d", i, j) ;
```

Write your answer here: \_\_\_\_\_

5. (2 points) Write a C-program to find the minimum value of a two dimensional array of integers.

6. (1 point) Plot  $f(x) = \frac{\sin(\pi x)}{x}$  as a function of  $x$ . Mark the maximum value, the place where this value is taken, and a representative set of  $x$  values (on either side of the origin) where  $f(x) = 0$ .

7. (1 point) The maximum of  $xe^{-x}$  is reached at  $x^* =$ \_\_\_\_\_.

8. (1 point) The sum of the squares of the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 4 \\ 4 & 5 & 1 \end{bmatrix}$$

is \_\_\_\_\_.

9. (1 point) The eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix}$$

are (write your answer in the space given below):

10. (1 point) Which of the following choices hold true for the vectors  $\left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \end{bmatrix} \right\}$ ?  
(You must tick all that apply.)

- Linearly independent
- Linearly dependent
- Neither linearly independent nor linearly dependent
- Orthogonal

11. (1 point) The rank of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

is \_\_\_\_\_.

12. (1 point) Let a biased coin be tossed  $n$  times in succession, with the probability of heads being  $p$ . The probability that all the tosses show the same face is \_\_\_\_\_.

13. (1 point) Let  $X$  and  $Y$  be two correlated random variables with means  $\mu_X$  and  $\mu_Y$ , respectively. The mean of the random variable  $X + Y$  is always

greater than  $\mu_X + \mu_Y$

less than  $\mu_X + \mu_Y$

$\frac{1}{2}\mu_X + \frac{1}{2}\mu_Y$

$\mu_X + \mu_Y$

(You must tick all that apply.)

14. (1 point) The number of undirected graphs, without multiple edges and without self loops, on a set of  $n$  labelled vertices is \_\_\_\_\_.

15. (1 point) The number of leaves in a complete binary tree of depth  $n$ , with the root node at depth 0, is \_\_\_\_\_.