Syllabus for MTech (AI) Written Test.

1. **Calculus**: Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Theorems of integral calculus, evaluations of definite and improper integrals. Taylor series. Partial derivatives, multiple integrals, directional derivatives.

2. **Linear Algebra and Matrix Analysis**: Vector space, basis, linear dependence and independence, matrix algebra, rank, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.

3. **Probability and Statistics**: Random variables. Uniform, normal, exponential, Poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional and joint probability, Bayes theorem. Correlation and regression analysis.

Sample Written Test Questions: Please see the next few pages for sample written test questions. Please note that no model answers will be provided.

Name: _____ Application Number: _____

Sample Questions for the Written Test.

1. Plot $f(x) = \frac{\sin(\pi x)}{x}$ as a function of x. Mark the maximum value, the place where this value is taken and a representative set of x values (on either side of the origin) where $f(x) = 0$.

2. The maximum of xe^{-x} is reached at $x^* =$ _____.

3. The function $f : \mathbb{R} \to \mathbb{R}$ is defined as

$$f(x) = \begin{cases} x^2 \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases},$$

where $\mathbb{R}$ is the set of real numbers. Which all of the following hold?

[You must clearly select all that apply to get credit]

A. f is continuous for all $x \in \mathbb{R}$

B. f is differentiable for all $x \in \mathbb{R}$

C. f is differentiable for all $x \in \mathbb{R}$ and the derivative is continuous for all $x \in \mathbb{R}$.

D. f is differentiable for all $x \in \mathbb{R}$ and the derivative equals 0 at an infinite number of points.

4. The sum of the squares of the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 4 \\ 4 & 5 & 1 \end{bmatrix}$$

is _____.

5. The eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 4 \end{bmatrix}$$

are (write your answer in the space given below):

6. Which of the following choices hold true for the vectors $\left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -4 \end{bmatrix} \right\}$? (You must tick all that apply.)
   A. Linearly independent
   B. Linearly dependent
   C. Neither linearly dependent nor linearly independent
   D. Orthogonal

7. The rank of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

   is _____.

8. Let a biased coin be tossed n times in succession, with the probability of heads being p. The probability that all the tosses show the same face is _____.

9. Let X and Y be two correlated random variables with means μ_X and μ_Y , respectively. The mean of the random variable $X+Y$ is always
   A. greater than $\mu_X + \mu_Y$
   B. less than $\mu_X + \mu_Y$
   C. $\frac{1}{2}(\mu_X + \mu_Y)$
   D. $\mu_X + \mu_Y$

   (You must tick all that apply)